PH.D. THESIS IN PHYSICS

Mesoscopic Superconductivity towards Protected Qubits

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This thesis has been submitted to the PhD School of
The Faculty of Science, University of Copenhagen
December 22, 2018
Abstract

This thesis presents results from experimental studies of three different approaches towards protected qubits based on novel semiconductor nanowires proximitized by an epitaxially grown aluminium shell.

Superconducting transmon qubits are promising candidates as building blocks in protected qubits based on quantum error correction. A Josephson junction formed in an InAs/Al core/shell nanowire exhibit a tunable Josephson energy achieved by an electrostatic gate depleting the carrier density of a semiconducting weak link region. We integrate an InAs/Al nanowire Josephson junction into a transmonlike circuit forming a gatemon. Embedding a gatemon into a microwave cavity, we observe a vacuum-Rabi splitting, and in the dispersive regime, we measure relaxation times up to 5 µs. Additionally, we demonstrate universal control of a two-qubit device.

Next, we exploit the non-cosinusoidal energy-phase relation of high-transmission, nanowire Josephson junctions in a superconducting interference device to form a 0-π qubit. The 0-π qubit can act as a fundamental building block for topologically protected qubits. Furthermore, voltage control of the semiconductor Josephson junctions creates a unique superconducting circuit allowing in situ tuning between widely different qubit regimes: transmon, flux, and 0-π qubit. Close to the 0-π regime, we observe enhanced lifetimes indicating protected qubit states.

Finally, it has been proposed to measure the direct coupling of two separated topological phases, required for control and readout of topological qubits, in a transmonlike circuit. We demonstrate the coherence of a transmon circuit based in InAs/Al nanowire Josephson junctions surviving up to magnetic fields of 1 T sufficient to enter a topological phase. Furthermore, we present a phenomenological model for coherent modes present at high magnetic fields coupling to transmon states.
Dansk Resumé

Denne afhandling præsenterer resultater fra eksperimentelle undersøgelser af tre forskellige teknikker til fejlbeskyttede kvantebits baseret på nye halvleder nanotråde proximitized af et epitaksielt påført aluminiumslag. Superledende transmon kvantebits er lovende kandidater til byggesten i fejlbeskyttede kvantebits baseret på kvante fejlkorrektion.

En Josephson kontakt, dannet i en InAs/Al kerne/skal nanotråde, har en justerbar Josephson energi kontrolleret af en elektrostatisk gate, som formindsker tætheden af ladningsbærer i en svag halvlederforbindelse. En gatemon dannes bed at integrerer en InAs/Al nanotråde Josephson kontakt i et transmonlignende kredsløb. Ved at indsætte en gatemon i en mikrobølgeresonator observerer vi en vakuum-Rabi splittelse og i spredningsregimet måler vi levetider op til 5 µs. Derudover demonstrerer vi universel kontrol af en doublekvantebitprøve.

Efterfølgende udnytter vi det ikke-cosinusformede energi-fase forhold mellem høj transmissions nanotråde Josephson kontakter i en superledende interferens enhed til at danne en $0-\pi$ kvantebit. $0-\pi$ kvantebiten kan fungere som en grundlæggende byggesten for topologisk fejlbeskyttede kvantebits. Spændingskontrol af halvleder Josephson kontakter skaber et unikt superledende kredsløb med $in\ situ$ tuning mellem vidt forskellige kvantebitregimer: transmon, flux, og $0-\pi$ kvantebit. Tæt på $0-\pi$-regimet observerer vi en indikation på fejlbeskyttede kvantebitstilstande i form af forbedret levetid.

Endelig er det blevet foreslået at måle den direkte kobling af to adskilte topologiske faser, som kræves til kontrol og udlæsning af topologiske kvantebits, i et transmonlignende kredsløb. Vi demonstrerer at kvantekohærens i et transmon kredsløb baseret på en InAs/Al nanotråde Josephson kontakt overlever op til magnetfelter på 1 T tilstrækkeligt for at tilgå topologiske faser. Desuden præsenterer vi en fænomenologisk model for tilstande observeret ved høje magnetfelter, som kobler til transmontilstande.
Acknowledgements

My work would not have been possible without the support from countless people. First, I would like to thank my supervisor Charlie Marcus. Charlie, it has been a pleasure to work under your guidance with the possibilities to redirect my research path to new topics and challenges during my studies. It has been a privilege to work in your laboratory both due to the high-end equipment and the open culture you facilitate leading to innumerable, enjoyable discussions and collaborations.

Next, I would like to thank Karl Petersson who has been my acting co-supervisor. Thank you for introducing me to the complicated world of high-frequency measurements and superconducting qubits. I have been happy to part of transmon team since its inception guided by your thoughtful approach and attention to detail.

I am thankful to everyone in the transmon team who have supported and contributed to my work. Special thanks to Lucas Casparis, who has contributed immensely both with measurements, fabrication, and ideas. Anders Kringhoj, thank you for the amazing teamwork and for always joining my off-schedule coffee breaks. I have had a plethora of great discussions on quantum control with Natalie Pearson but somehow we manage to never agree on the details of software architectures. Also a big thanks to Rob McNeil for always bringing a smile as well as all the fabrication you have done for me. Oscar Erlandsson, thank you for selflessly letting me be part of measurements on samples you fabricated.

I want to thank Andrew Higginbotham who taught me the ropes of experimental condensed matter physics. Also thanks to Ferdinand Kuemmeth for always providing new perspectives to measurements and always having new curious thought experiments. Misha Gershenson, thank you for sharing your expertise and many discussions on designs and measurements of novel experiments. I would like to thank Matthias Christandl, Gorjan Alagic, and Héctor Bombín for answering countless questions in discussions and journal clubs on quantum information science. In the latter part of my Ph.D., I got the opportunity to work on topological materials together with the cQED qubit team in Microsoft. I want to thank Angela Kou and everyone else in the Delft team for a close collaboration. Also a big thanks to Bernard van Heck and the theory team in Santa Barbara for answering many unreasonably hard questions about topological materials.

My first years in QDev wouldn’t have been nearly as pleasant without Christian Olsen. Thank you for the many late hours at QDev juggling interesting research and less interesting coursework. We were fortunate enough to also share the office with great
officemates Morten Hels and Jerome Mlack. Henri Suominen and Giulio Ungaretti thank you for many enjoyable lab dinners as well as many Wednesday traditions. Also thanks to Shivendra Upadhyay inviting me to your wedding. I would also like to thank Sven Albrecht for a nice trip to Austin. Many thanks to everyone in QDev who makes it such a great place to work and learn.

Of course, a laboratory is non-functioning without the great support from technicians and secretaries who are really making the research possible. Big thanks to Shivendra Upadhyay and Dorthe Bjergskov and everyone else making this possible. Also thanks to the QCoDeS team in Copenhagen both for teaching me how to code and providing the software required in the lab.

During my Ph.D., I had the privilege of visiting Will Oliver’s Lab at MIT for three months. I would like to thank Will for the opportunity to visit and being immediately trusted with ongoing measurements. Also thanks to all my fellow students and researches in the group which made my stay incredibly enjoyable and educating. Especially, thanks to Morten Kjærgaard for welcoming me to Boston and the many elucidating coffee discussions. I hope I will have the opportunity for many more visits in the future.

Lastly, I would like to thank my family who has supported me throughout even when my studies seemed to take precedence over everything else.
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Chapter 1

Introduction

The continued digital transformation of society since the invention of the transistor [1, 2] and the integrated circuit [3] has been driven by Moore's law stating that the number of transistors per area will grow exponentially [4]. However, as Moore's law is coming to an end, due to the size of transistors reaching physical limitations, several problems are still thought intractable even on tomorrow's supercomputers. One such problem is the accurate simulation of large quantum systems with applications in drug development, chemical reactions, materials science as well as general understanding of nature. Realizing the potential of quantum simulations, Richard Feynman proposed a new type of computer, a quantum computer, capable of simulating nature [5]. Since then several concrete algorithms have been proposed with widespread applications for molecule simulations [6, 7], machine learning [8], and database searching [9]. Most notably in 1994, Peter Shor published a quantum algorithm efficiently breaking RSA encryption [10] exemplifying the widespread influence of a quantum computer on society.

A digital quantum computer is based on replacing the classical bit with a quantum bit (qubit), a quantum mechanical two-level system. Qubits allow the information itself of a computation to be in entangled superposition states opening new possibilities in algorithms. A modest 300 qubit quantum computer can work with $2^{300}$ different states simultaneously - that is more states than there are atoms in the universe! While qubits allow new algorithms they also introduce new sources of errors. The challenge lies in building a qubit decoupled from any noise but easily manipulated to perform computations. Several qubit platforms are actively being investigated: ion traps [11–13], superconducting qubits [14–18], spin-qubits [19, 20], and topological material [21–25] among many others. Qubit performance continues to improve dramatically each year, but orders of magnitude better qubits are required for a fully functional quantum computer.

The next milestone is the development of topologically protected qubits in which the control mechanisms are topologically different from noise sources. The goal is to encode a qubit into a non-local degree of freedom which is exponentially decoupled from local noise sources as the system size is increased. Topological protection can be achieved via quantum error correction [26–28], passive quantum error correction [29], or topological materials [30]. This thesis investigates each approach to protected qubits in mesoscopic super-
conducting devices incorporating hybrid InAs-Al semiconductor-superconductor nanowires [31].

The basic idea of quantum error correction is to confine the Hilbert space of a multi-qubit system to a non-local subspace by local measurements. Any local noise is then detectable from the eigenvalues of the local measurements protecting the non-local subspace. State-of-the-art superconducting qubits are rapidly approaching a quality and quantity sufficient for quantum error correction [32–34]. Mesoscopic, condensed matter systems are promising candidates for error-corrected qubits due to the potential for scalability by leveraging existing fabrication technology from the semiconductor industry. In this thesis we investigate hybrid semiconductor-superconductor qubits, gatemons, combining field effect tunable semiconductors with dissipationless superconductors. Superconducting qubits are anharmonic resonant circuits formed by a Josephson junction shunted by a capacitor. In gatemons, the Josephson junctions are created from proximitized semiconductor materials allowing \textit{in situ} voltage tuning of qubit parameters.

Passive quantum error correction similarly relies on confining a subspace spanned by non-local degrees of freedom. However, instead of confining the subspace by active measurements a Hamiltonian is designed to form degenerate non-local ground states inherently. Each measurement of an error correcting code is replaced by an energy gap in the Hamiltonian which acts as a passive measurement by the system itself. The Hamiltonian will have non-local, degenerate ground states isolated from local noise due to an energy gap. A fundamental element required to engineer such systems are qubits with degenerate ground states. Ongoing investigations rely on superconducting circuits with insulator junctions [35–37]. Hybrid semiconductor-superconductor junction introduces a new circuit element similar to insulator junction but with crucial differences due to the high mobility of semiconductors. We explore simple mesoscopic circuit architectures utilizing high-transmission junctions for protected qubits.

The specific material combination of one-dimensional InAs/Al, which has a strong spin-orbit coupling and superconductivity has long been investigated as a topological material hosting non-local excitations. For topological materials, the non-local nature of excitations is achieved on the microscopic level of electron-electron interactions. In this thesis we develop a superconducting circuit, taking advantage of control techniques from superconducting qubits, designed to probe the coupling of topological phases essential for control and readout of topological qubits. We demonstrate that coherent superconducting circuits be realized with control circuitry and high magnetic fields required for topological qubits in InAs/Al nanowires.

This Ph.D. thesis is written as part of the so-called integrated (4+4)PhD program at University of Copenhagen. Thus, parts of Chapter 3 and 5 presented in this thesis also appear in the authors master thesis (reference [38]). We note that this practice is consistent with the spirit and regulations of the integrated Ph.D. program.
1.1 Outline

The outline of this thesis is as follows:

In Chapter 2 we introduce the basics of qubits and quantum information as well as the theory of protected qubits. The theory of mesoscopic harmonic oscillators and artificial atoms in superconducting circuits, circuit quantum electrodynamics, is presented in Chapter 3. Furthermore, the semiconductor-superconductor Josephson junction and its characteristics are introduced. Chapter 4 gives a description of the fabrication flow for each sample as well as an overview of the experimental setup and measurement techniques. In Chapter 5 the development of the gateon qubit is presented, and single and two-qubit operations are benchmarked. The first steps towards protected qubits with passive quantum error correction based on high-transmission Josephson junction are presented in Chapter 6. We show that degenerate qubits can be formed with signatures of protected states. Lastly, in Chapter 7 we introduce a high magnetic field compatible superconducting qubit for detection of topological phases. Chapter 8 gives an outlook on the field of experimental quantum computing.
CHAPTER 1. INTRODUCTION

1.2 Publications

The work during the thesis project has resulted in the following publications.


* These authors contributed equally.
Chapter 2

Theory of Quantum Computing

In this chapter, we first introduce the mathematical concept of a qubit, qubit operations, and a simple quantum algorithm. Next, we introduce each of the different approaches to topological protection necessary for practical quantum computing.

2.1 Quantum Bits

A classical bit is some physical system that can take two values commonly denoted as 0 and 1. In computers, calculations are performed on bits of information represented by low or high voltage with a threshold voltage defining if it is 0 or 1.

A quantum bit, or qubit, is some quantum system that has two linearly independent states commonly denoted as $|0\rangle$ and $|1\rangle$. While a bit can only be in two states, the state of a qubit, $|\psi\rangle$, can be any linear combination of $|0\rangle$ and $|1\rangle$:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \left(\begin{array}{c} \alpha \\ \beta \end{array}\right), \quad (2.1)$$

where $\alpha$ and $\beta$ are complex numbers normalized by $|\alpha|^2 + |\beta|^2 = 1$. The state of an isolated single qubit can be parametrized by three real numbers

$$|\psi\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2}|0\rangle + e^{i\phi} \sin \frac{\theta}{2}|1\rangle\right). \quad (2.2)$$

As the global phase of the state, $\gamma$, is not observable in a single qubit system we can ignore this factor. We are left with two numbers $\theta$ and $\phi$ which can be visualized as a point on a sphere - the Bloch sphere. Figure 2.1A visualizes the Bloch sphere with state $|\psi\rangle$ marked as a point. The Bloch sphere is an incredibly powerful tool for understanding single-qubit operations. An operation $U$ applied to the state $|\psi\rangle$ can be represented as a rotation (up to a global phase) of the qubit state on the Bloch sphere [Figure 2.1B].

The qubit state can be at any point on the Bloch sphere but when measured the
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qubit will only take one of two values. A projective measurement of the eigenvalue of \( \hat{\sigma}_z = |0\rangle\langle 0| - |1\rangle\langle 1| \) will yield +1 with probability |\( \alpha \)|^2 and -1 with probability |\( \beta \)|^2.

The Hamiltonian describing the time-evolution of the qubit state is ideally given by:

\[
\hat{H} = 0,
\]

i.e. the qubit state is a constant in time\(^1\). A qubit operation can be described as a controlled time-evolution by changing the Hamiltonian. Without loss of generality we can decompose the Hamiltonian into three independent terms:

\[
\hat{H} = \hbar \frac{\Omega_x(t)}{2} \hat{\sigma}_x + \hbar \frac{\Omega_y(t)}{2} \hat{\sigma}_y + \hbar \frac{\Omega_z(t)}{2} \hat{\sigma}_z,
\]

where \( \Omega_i(t) \) describes the applied operation and \( \hat{\sigma}_i \) are Pauli matrices:

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

A rotation around the \( x \)-axis shown in Figure 2.2 can be induced by setting \( \Omega_x(t) = \Omega \) while keeping \( \Omega_y(t) = \Omega_z(t) = 0 \). The time evolution of the qubit state is then given by:

\[
R_x(\Omega t)|\psi\rangle = e^{-i\Omega t \hat{\sigma}_x}|\psi\rangle = \begin{bmatrix} \cos \frac{\Omega t}{2} & -i \sin \frac{\Omega t}{2} \\ -i \sin \frac{\Omega t}{2} & \cos \frac{\Omega t}{2} \end{bmatrix} |\psi\rangle = \begin{bmatrix} \cos \frac{\Omega t}{2} & -i \sin \frac{\Omega t}{2} \\ -i \sin \frac{\Omega t}{2} & \cos \frac{\Omega t}{2} \end{bmatrix} |\psi\rangle.
\]

After a time \( t \), the qubit will have rotated an angle \( \Omega t \) around the \( x \)-axis of the Bloch sphere. For example, for \( \Omega t = \pi \) the rotation applies the operation \(-i\hat{\sigma}_x|\psi\rangle = -iX|\psi\rangle\), where \( X = \sigma_x \) is the conventional notation in quantum computer science. Similarly, rotations can be induced around \( y \) and \( z \) by \( \Omega_y \) and \( \Omega_z \) respectively. Practically it is

\[^1\text{In practice } \hat{H} = 0 \text{ is often described in a rotating frame of reference.}\]
sufficient to implement control of just two orthogonal axes as any single-qubit operation can be decomposed as $U = e^{i\alpha}R_x(\beta)R_y(\gamma)R_x(\delta)$, where $\alpha, \beta, \gamma,$ and $\delta$ are real numbers [39].

Multi-qubit systems have many of the same properties as a single qubit. The system state now has four linearly independent states often represented in the computational basis as

$$|\psi_2\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle,$$

(2.7)

where $\alpha_{ij}$ are complex numbers. Unfortunately, a visual representation of a two-qubit state would require a 7-dimensional space. Universal control of a two-qubit system can be achieved with universal single-qubit gates and one entangling two-qubit gate [40]. Practically, this is an incredibly important result as only a single type of qubit-qubit coupling needs to be designed and optimized. The specific gate, being implemented, depends on the details of the system.

A common group of gates, which plays an essential role for protected qubits, is the Clifford group. The group of Clifford gates is generated by the gate set:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, \text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

(2.8)

where CNOT is the controlled-not gate. The controlled-not gate, also sometimes referred to as CX (controlled X), performs an X gate on a target qubit dependent on the state of a control qubit, e.g., $\text{CNOT}_{01}|10\rangle = |11\rangle$ where subscript 01 refers to index 0 and 1 of control and target qubit respectively. The Clifford group for a single qubit can be visualized as any gate that preserves the octahedron of the Bloch sphere as shown.
in Figure 2.3. There are 24 gates in the single-qubit Clifford group - the number of orientations the octahedron can take. The two-qubit Clifford group has 11,520 elements \[41\].

Having introduced qubits and gates we can now look at a simple non-trivial circuit taking advantage of the quantum mechanical nature: teleportation of quantum information. Figure 2.4 depicts a 3-qubit circuit which teleports the quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ of qubit 1 onto qubit 3 without gaining any information about the state. The circuit consists of single-qubit gates and CNOT gates as well as classical information from measurement results depicted as double lines.

First qubit 2 and 3 are put in a two-qubit entangled state, a Bell state, such that the total state of the system at $|\psi_1\rangle$ is

$$|\psi_1\rangle = \text{CNOT}_{23} H_2 |\psi\rangle |00\rangle$$

$$= |\psi\rangle \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$= \alpha \frac{|000\rangle + |011\rangle}{\sqrt{2}} + \beta \frac{|100\rangle + |111\rangle}{\sqrt{2}},$$

where the subscripts of the gate refers to which qubit(s) the gate is applied to. Next qubit 2 is entangled with qubit 1 leading to the system state

$$|\psi_2\rangle = H_1 \text{CNOT}_{12} |\psi_1\rangle$$

$$= \frac{1}{2} [\alpha(|000\rangle + |100\rangle + |011\rangle + |111\rangle) + \beta(|010\rangle - |110\rangle + |001\rangle - |101\rangle)]$$

$$= \frac{1}{2} [|00\rangle (\alpha|0\rangle + \beta|1\rangle) + |10\rangle (\alpha|0\rangle - \beta|1\rangle) + |01\rangle (\alpha|1\rangle + \beta|0\rangle) + |11\rangle (\alpha|1\rangle - \beta|0\rangle)]$$

$$= \frac{1}{2} [|00\rangle |\psi\rangle + |10\rangle Z|\psi\rangle + |01\rangle X|\psi\rangle + |11\rangle XZ|\psi\rangle].$$

By measuring the states of qubit 1 and 2, the system will collapse into one of the four possible states in equation (2.13) leaving qubit 3 in state $|\psi\rangle$ up to a single qubit gate. Depending on the results of the two measurements correction gates are applied to qubit...
3 to complete the teleportation of an unknown state.

Teleportation is a simple algorithm with only six gates and two measurements making up the full algorithm, which state-of-the-art qubits can readily implement and run with very high efficiency [42]. However, we are far from computing larger and useful computations containing orders of magnitude more gates due to non-ideal operations.

### 2.2 Quantum Error Correction

The classical repetition code is a simple example of error correction. It takes a single bit of information and encodes it in three physical bits $\bar{1} = 111$ and $\bar{0} = 000$, where the bar notation represents the encoded (logical) information. The physical representations of the logical information, here 111 and 000, are called the codes codewords. If a bit-flip error happens on a single bit, one can still decode the encoded information by taking a majority vote of all the bits. However, if two bit-flip errors happen in different bits the decoding by a majority vote will give the wrong answer. For bits with error rate $\rho$, the three-bit repetition code will have an error rate of order $\rho^2$ assuming the errors are independent.

One cannot directly use the same type of error correction for qubits. A repetition code relies on copying the information of one bit to several bits - a process that is impossible for qubits due to the no-cloning theorem [39]. Furthermore, to detect an error in the repetition code one has to measure all the bits which would collapse any superposition state of the qubit. Instead, quantum error-correcting codes work by encoding the qubit state in a multi-qubit degree of freedom combining many qubits to an effective two-level system [39, 43]. Error detection is achieved by measuring a specific set of multi-qubit operators, also called the codes stabilizers\(^3\), rather than single-qubit states.

The stabilizer formalism is incredibly powerful for describing error-correcting codes [44]. A quantum state $|\psi\rangle$ is stabilized by a stabilizer $S$ if $S|\psi\rangle = |\psi\rangle$. For a two-qubit state, an example of a stabilizer could be $X_1X_2$ which stabilizes any linear combination of the two quantum states $\left\{\frac{|00\rangle + |11\rangle}{\sqrt{2}}, \frac{|01\rangle + |10\rangle}{\sqrt{2}}\right\}$. Here a single stabilizer $X_1X_2$ uniquely defines a subspace of the two-qubit Hilbert space without having to specify eigenstates.

\(^3\)Other non-stabilizer quantum error-correcting codes exist but are not covered here.
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spanning the subspace. The formalism efficiently describes quantum error-correcting codes whose quantum states becomes very long and unintuitive written in the computational basis.

Returning to the repetition code, we can describe the quantum version using stabilizers. The quantum repetition code is defined by the stabilizer group generated by $g_1 = \{Z_1Z_2I, I_1Z_2Z_3, I_1I_2Z_3, Z_1I_2Z_3\}$. The codewords of the code are then given by the quantum states stabilized by $S$: $\{|000\}, \{|111\}\}$. The protected qubit state can be written as:

$$|\bar{\psi}\rangle = \alpha |000\rangle + \beta |111\rangle.$$  

(2.14)

In general, a quantum code made of $n$ qubits with $m$ generators can encode $n - m$ protected qubits$^4$. Similarly to the classical version, the code can detect a single bit-flip error on any qubit. By the definition of stabilizers, we can measure the eigenstate of any stabilizer without disturbing the encoded information: $S_i |\bar{\psi}\rangle = +1 |\bar{\psi}\rangle$. Error detection can be performed by measuring the eigenvalues of stabilizers of the code. It is sufficient to measure a set of generators of the stabilizer group as the eigenvalues of other stabilizers can be computed from these. The set of measured eigenvalues is called the error syndrome.

Assume the code had a bit-flip error $X_1$ leaving the code in state

$$X_1|\psi\rangle = \alpha |100\rangle + \beta |011\rangle.$$  

(2.15)

Measuring the error syndrome we find eigenvalues $g_1 X_1 |\psi\rangle = -X_1 g_1 |\psi\rangle = -X_1 |\psi\rangle$ and $g_2 X_1 |\psi\rangle = X_1 g_2 |\psi\rangle = X_1 |\psi\rangle$ revealing an error as a change in the eigenvalue of $g_1$. Two errors could have caused the error syndrome $\langle g_1 \rangle = -1$ and $\langle g_2 \rangle = 1$: $\{X_1, X_2 X_3\}$. The stabilizer formalism allows one to find this set simply by analyzing possible errors which anti-commutes with $g_1$ and commutes with $g_2$. The most likely error to have happened is the single-qubit error $X_1$, which can be recovered by applying a recovery gate $X_1^\dagger = X_1$ to the system.

In general, a stabilizer code defined by stabilizers $S$ can correct any error $E_j$ from the set $E$ if

$$\forall E_j, E_k \in E; \exists S \in S; E_j^\dagger E_k S E_k^\dagger E_j \notin S \text{ or } E_j^\dagger E_k \in S.$$  

(2.16)

One needs to consider two error operations $E_j^\dagger$ and $E_k$ as the recovery gate found from the error syndrome is itself an error. Effectively the code needs to be able to detect two errors simultaneously to allow error correction of a single error. Detect that an error happened and detect which recovery gate will remove the error.

We have described how a single bit-flip error can be detected and corrected by the repetition code. However, a bit-flip error is just one of an infinite number of errors that

$^4$This intuitively makes sense as each generator $g_i$ confines the state $|\psi\rangle$ to the part of the Hilbert space which has $g_i |\psi\rangle = |\psi\rangle$ - thereby excluding the other half with $g_i |\psi\rangle = -|\psi\rangle$.  

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can happen to a qubit. An error can be a small rotation of the qubit or a complete entanglement with an uncontrolled part of the environment. It is not trivial that error correction of a quantum state is even possible - how does one measure which error happened from a continuous set of errors? Quantum error correction is made possible by the fact that a superposition state collapses into just one state when measured effectively reducing the set of possible errors from infinite to finite.

Any single-qubit error $E$ can be expanded into a linear combination of Pauli errors

$$E = e_I I + e_X X + e_Y Y + e_Z Z,$$  \hspace{1cm} (2.17)

where $e_i$ is the probability for error $i$ happening on the qubit. The qubit state after an error is $E|\psi\rangle = e_I |\psi\rangle + e_X |\psi\rangle + e_Y |\psi\rangle + e_Z |\psi\rangle$. Error detection of the Pauli errors $X,Y,Z$ will detect the error syndrome which will collapse the superposition into just one of the four options. That is a quantum error correcting code being able to correct Pauli errors can correct any single-qubit error as the detection discretizes the error.

Returning to the quantum repetition code defined by generators \{Z_1, Z_2, Z_3\} which can correct bit flip errors of the type \{X_1, X_2, X_3\}. To allow the repetition code to correct for any single-qubit error one has to expand it to detect errors $Z_i$ also (it is enough to correct for $X_i$ and $Z_i$ as $Y_i = X_i Z_i$). This code is known as the Shor code from the inventor Peter Shor [45]. First one realizes from symmetry that a repetition code with generators \{X_1X_2, X_2X_3\} can correct the error set $Z_1, Z_2, Z_3$ - also called phase-flip errors. To correct for both $X$ and $Z$ errors the repetition code is concatenated. Three bit-flip repetition codes each able to correct bit-flip errors is used as single qubits in a phase-flip repetition code. In total nine qubits are combined with four repetition codes defined by eight stabilizers shown in Table 2.1. The first six stabilizers define the three separate bit-flip repetition codes. The last two combines these three codes using the logical operators of each in the stabilizers. It is easily shown that any two single-qubit errors $E_j E_k$ either anti-commutes with at least one stabilizer or is itself a stabilizer fulfilling Equation (2.16). E.g. $Y_2 Z_3$ anti-commutes with the stabilizer $g_1 = Z_1 Z_2$.

The extension of the repetition code to the Shor code exemplifies the power of the

<table>
<thead>
<tr>
<th>$g_1$</th>
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<tr>
<td>$g_2$</td>
<td>$Z_2Z_3$</td>
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<tr>
<td>$g_3$</td>
<td>$Z_4Z_5$</td>
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<td>$g_4$</td>
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<td>$g_5$</td>
<td>$Z_7Z_8$</td>
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<td>$g_6$</td>
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<tr>
<td>$g_7$</td>
<td>$X_1X_2X_3X_4X_5X_6$</td>
</tr>
<tr>
<td>$g_8$</td>
<td>$X_2X_5X_6X_7X_8X_9$</td>
</tr>
</tbody>
</table>

Table 2.1: Stabilizers defining the Shor code which is formed from four repetition codes.
stabilizer formalism. The quantum states of the codewords are given by:

\[
|0\rangle = \frac{(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}}
\]

\[
|1\rangle = \frac{(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}}
\] (2.18)

The codewords are superpositions of eight states in the computational basis. Stabilizers allow us to define the full code from just eight stabilizers while the codewords are large superposition states.

The Shor code can be expanded to detect larger and larger error sets by increasing the number of qubits. However, other codes have shown better performance for scalability. The most exciting codes currently being investigated both theoretically and experimentally are topological codes. These are codes that are defined by local stabilizers with global logical operators. One such example is the surface code [26] shown in Figure 2.5. Qubits are shown as open circles while four-body stabilizers are shown in green and yellow for $Z_{i,1}Z_{i,2}Z_{i,3}Z_{i,4}$ and $X_{i,1}X_{i,2}X_{i,3}X_{i,4}$ respectively. The quantum information is encoded as a global degree of freedom while the stabilizers are local. Any local error (a row of errors not extending half-way across the code) can be detected. It follows that by making the code larger, it can correct for larger errors.

The surface code has received a great deal of experimental attention due to the low error-threshold and relatively simple implementation [46]. Only nearest neighbour couplings of qubits and Clifford gates are necessary to fully implement a patch of surface code. The threshold is the maximum error rate at which making the code larger will extend the lifetime of the encoded qubit. Estimating thresholds is very dependent on assumptions about the physical implementation, the error model, and the syndrome decoder. A code can be implemented in any qubit system, although the implementation of stabilizer measurements can vary drastically from system to system. Naturally, this also affects the error rates of stabilizer measurement which effects the error threshold. Syndrome decoding is an interesting and complicated topic on its own. To correct for
errors, syndrome decoding collects all the local stabilizer measurements in classical logic and performs a global computation to find the most likely set of errors. Adding that the stabilizer measurements themselves can be faulty one has to do several stabilizer measurements, in between which new errors can happen. It turns out the problem of optimal syndrome decoding becomes an intractable problem which a classical computer cannot efficiently solve [47]. Fortunately, new algorithms achieving sufficient (although not optimal) performance has been developed so that we don’t need a quantum computer to be able to error correct a quantum computer [48].

2.3 Passive Error Correction

In the previous section, we introduced stabilizer codes. These codes rely on measurements of error syndromes to detect, decode, and correct the errors. This potentially adds a massive overhead in control electronics and computation time. An alternative path is to implement passive quantum error correction [29]. The basic idea is to form a physical system that effectively separates stabilizer eigenstates in energy. A Hamiltonian implementing the Shor code is given by

\[
\hat{H} = -\frac{\Delta}{2} \sum_{S_i \in S} S_i,
\]

where \(S\) is the group of stabilizers with generators given in Table 2.1. In this Hamiltonian, the ground state is doubly degenerate separated by an energy gap \(\Delta\) from all other eigenstates of the system.

To build such a system one needs to construct degenerate qubits with specifically engineered coupling terms to form an effective Hamiltonian of the form in Equation (2.19). The qubits need to be defined by degenerate two-level systems as any energy splitting between the qubit states will introduce undesired single-qubit terms in the Hamiltonian. B. Doucôt and L. B. Ioffe describe in [29] a possible system for protected qubits based on passive quantum error correction. The basic building block in the system is a superconducting circuit element with the degenerate ground states given by a superconducting phase difference across the circuit of \(\pm \frac{\pi}{2}\) [Figure 2.6A]. We can describe these two states with Pauli operators where the states \(|\pm \frac{\pi}{2}\rangle\) are eigenstates of \(\sigma_z\).

To form a protected Hamiltonian, which is described as a sum of stabilizers of the repetition code, one first places several qubits in an array as shown in Figure 2.6B. The total phase across the array will be \(\gamma = \sum_j \sigma_{z,j} \mod 2\pi\). However, as \(\gamma\) can take only one of two values, it can be calculated as a product \(\gamma = \prod_j \sigma_{z,j} \text{for an odd number of elements and } \gamma = \prod_j \sigma_{z,j} + 1 \text{ for an even number of elements, where } P = \pm 1\) depending on the length of the array. The sign of \(P\) changes for every two qubits added to the array: \(P = 1\) for arrays of length 1, 2, 5, 6, 9, 10, ... and \(-1\) otherwise. Here \(\prod_j \sigma_{z,j}\) in a Hamiltonian acts similar to stabilizers of the Shor code \(g_7\) and \(g_8\) in Table 2.1. In Figure 2.6C multiple arrays are connected enforcing a common phase across the arrays. Due to the common phase, there is a high energy cost associated to a single qubit flipping.
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Figure 2.6: A Qubits with degenerate ground states $|\phi = \pm \pi/2\rangle$, where $\phi$ is the phase difference across the circuit. Each ground state can be described as an eigenstate of $\sigma_z$. B A row of elements with a total phase $\gamma = \sum_j \sigma_{z,j} \mod 2\pi$. C Multiple rows are coupled to form a protected qubit with a common phase, $\gamma$, across the rows.

$\sigma_z$ eigenstate - like an error correcting code detecting X-errors by measuring $\prod_i Z_i$ along arrays of qubits.

To also create protection against phase flips the Hamiltonian needs to contain terms of the form $\sigma_{x,j}\sigma_{x,j+1}$ similar to the two-body stabilizers $g_1 - g_6$ in Table 2.1. For superconducting qubits, a coupling of the form $\sigma_{x,j}\sigma_{x,j+1}$ is generated by a small charging energy between subsequent qubits in an array [29]. In combination with the terms of $\prod_j \sigma_{z,j}$ from above this can form a Hamiltonian mimicking the error protection of the Shor code. With clever circuit layouts other error correcting codes, such as the surface code, can also be implemented as Hamiltonians.

Passive error correction has the advantage that no costly syndrome analysis has to be performed as errors effectively are gapped out by the system. Reducing the amount of classical control needed for error correction frees up encoded qubit to compute actual quantum algorithms. While the error correction is implemented by connecting many degenerate qubits, it is still possible to probe a single qubit, or rhombus [36], at a time to gain information of the basic building blocks of the code. When the single qubit behaves as expected several can be connected to add error correction to the system. The difficulty lies in the fact that an inherently protected qubit is increasingly difficult to measure and control.

2.4 Topological Material

The goal of error correction, both passive and active, is to remove errors from a non-perfect system. What if instead nature provided a topological material with degenerate, non-local ground states protected from errors by an energy gap? A. Kitaev proposed this idea in [30] as an alternative path to a high-fidelity quantum computer. The model proposed, known as the Kitaev chain, is a one-dimensional chain of electrons at sites $i$ described by the Hamiltonian

$$H = -\mu \sum_i \hat{c}_i^\dagger \hat{c}_i - \frac{t}{2} \sum_i \left( \hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_{i+1}^\dagger \hat{c}_i \right) - \frac{\Delta}{2} \sum_i \left( e^{i\phi} \hat{c}_i^\dagger \hat{c}_{i+1} + e^{-i\phi} \hat{c}_{i+1}^\dagger \hat{c}_i \right), \quad (2.20)$$
where $\hat{c}^\dagger$ and $\hat{c}$ are fermion creation and annihilation operators respectively, $\mu$ is the chemical potential, $t$ is the nearest-neighbor hopping term, and $\Delta$ is a superconducting electron-electron coupling term with phase $\phi$. A detailed review of the properties of this Hamiltonian is given in [49]. Here we will focus on two specific cases: a trivial case with $\mu < 0$ and $\Delta = t = 0$ and a topological regime with $\mu = 0$ and $\Delta = t \neq 0$.

To understand the physics of the two regimes, it is beneficial to rewrite the Hamiltonian using Majorana fermion operators. A Majorana fermion is a particle which is its own anti-particle and follows fermion anti-commutation relations.

$$\{\hat{\gamma}_\alpha, \hat{\gamma}_\beta\} = 2\delta_{\alpha\beta}. \quad (2.21)$$

A single electron can be decomposed into two Majorana fermions.

$$\hat{c}_i = \frac{e^{-i\theta}}{2}(\hat{\gamma}_{B,i} + i\hat{\gamma}_{A,i}),$$
$$\hat{c}^\dagger_i = \frac{e^{i\theta}}{2}(\hat{\gamma}_{B,i} - i\hat{\gamma}_{A,i}). \quad (2.22)$$

where $\theta$ is a global phase. Setting $\theta = \phi/2$ Equation (2.20) can be written as

$$H = -\frac{\mu}{2} \sum_i (1 + i\hat{\gamma}_{B,i}\hat{\gamma}_{A,i}) - \frac{t + \Delta}{4} \sum_i i\hat{\gamma}_{B,i}\hat{\gamma}_{A,i+1} - \frac{\Delta - t}{4} \sum_i i\hat{\gamma}_{A,i}\hat{\gamma}_{B,i+1}, \quad (2.23)$$

where $P_i = i\hat{\gamma}_{B,i}\hat{\gamma}_{A,i} = \pm 1$ is the parity of the fermion at site $i$ defined as $-1$ for vacuum
and +1 for filled fermion. In the regime of $\mu < 0$ and $\Delta = t = 0$ the latter two terms disappear leaving a fermion counting term as shown in Figure 2.7A. The Hamiltonian has a single ground state given by vacuum. Any excitation is gapped by an energy cost $\mu$ for introducing a fermion in the system.

Setting the $\mu = 0$ we can understand the last two terms of the Hamiltonian as inter-site Majorana couplings depicted in green and blue in Figure 2.7B. In the case of $\Delta = t$ only one inter-site coupling is non-zero. Coupled Majorana fermions $\hat{\gamma}_{B,i}$ and $\hat{\gamma}_{A,i+1}$ form electron degrees of freedom with an energy $\Delta + t$ for each filled electron state. The bulk of the Kitaev chain is again described by a vacuum ground state now with energy gap $\Delta + t$. However, this leaves a single, uncoupled Majorana fermion at each end of the chain. These can be described by a non-local electron with $\hat{c}_M = \frac{1}{2}(\hat{\gamma}_{A,1} + i\hat{\gamma}_{B,N})$. As $\hat{c}_M^\dagger \hat{c}_M$ is not present in the Hamiltonian, they form a zero-energy two-level system which is protected from the noise due to its non-local nature. The appearance of uncoupled Majorana states at each end originates from a change in the topology of the material. This enforces a stability of the states to small changes of the parameters $\mu$, $t$, and $\Delta$. It is not only the single point $\mu = 0$ and $\Delta \neq t$ in phase space that is topological but a surrounding domain [49].

If such states can be created and controlled in nature, one can take advantage of the inherent protection afforded by the topological material. One challenge for an experimental realization is that the Kitaev chain is spin-less. If formed by an electron system with Kramer’s degeneracy there will be two Majorana states at each end - one for each spin flavor. Any spin-orbit interaction will couple these states forming local electron states breaking the protection. Lutchyn et al. and Oreg. et al. proposed in 2010 [50, 51] a solution to this problem based on a one-dimensional nanowire with spin-orbit coupling, placed in a magnetic field, and strongly coupled to superconductor as shown in Figure 2.8A. The Kramer’s degeneracy is lifted due to the combination of spin-orbit coupling and Zeeman splitting while the superconductivity provides the electron-electron coupling present in the Kitaev chain. The spin-orbit coupling can be understood in momentum space of the electron bands in the nanowire as a separation of spin-bands shown in red and blue in Figure 2.8B. To freeze the spin-degree of freedom at the Fermi surface a magnetic field is added to open a gap between the two parabolas at $k = 0$. With the chemical potential placed in the gap, only one electron band is present at the Fermi surface effectively forming a spin-less system.

Topological materials with non-local, protected degrees of freedom offer a unique path to high-fidelity qubits. The challenge lies in creating the topological material, and maybe even more challenging, do it in such a way that the protected qubit can be both controlled and measured.

2.5 Fault-Tolerant Quantum Computing

The previous sections described three different paths to protected qubits based on encoding qubits in non-local degrees of freedom. However, this alone will only form quantum memory while a quantum computer needs to perform computations. Here we will briefly
touch on the subject of fault-Tolerant quantum computing to put in perspective the challenges still ahead of us\textsuperscript{5}.

Focusing on a protected qubit with quantum error correction, assume that a single-qubit error happens before a CNOT gate as in Figure 2.9. For certain errors, a single qubit error before the two-qubit gate is equivalent to having two-qubit errors after it. The single error got multiplied by the operation posing a huge problem for error correction. If qubit operations are performed naively, a single qubit error can potentially spread throughout the code corrupting the protected information. Any control, including syndrome measurements, has to be implemented in a fault-tolerant manner, i.e. any error before an operation should remain correctable after the operation. This effectively limits the possible operations that can be performed on an encoded qubit to a finite set dependent on the specific code. The same limitations hold for fault-tolerant gate sets in topological materials and with passive error correction.

In all cases of topological quantum computers actively being pursued the set of possible gates is either the Clifford group or a subset thereof. However, the Clifford gate set is not a universal and is therefore not enough to build a universal quantum computer. In fact such a limited quantum computer has been proven to be no better than a classical computer. The solution is to add one more allowed gate to the quantum computer - the gate $T = R_Z(\frac{\pi}{4})$ plus the Clifford group is sufficient for universal quantum computing\textsuperscript{6}.

How then to perform fault-tolerant $T$-gates? One way is to perform magic state distillation of $T$-gates \cite{52, 53}. Magic state distillation is an algorithm which produces high fidelity $T|0\rangle$ states from a many noisy $T|0\rangle$ states. With this a non-fault-tolerant version of a $T$-gate is sufficient for quantum computing. The downside is that some estimates indicated that a quantum computer will have an enormous overhead just creating $T$-gates

\textsuperscript{5}In most cases theoretical solutions have been found but whether they are experimentally practical on an encoded qubit remains to be seen.

\textsuperscript{6}The set of Clifford gates and the $T$-gate cannot strictly perform all gates. Rather it is a dense gate-set which can perform gates arbitrarily close to any gate - analogous to a rational number being arbitrarily close to any real number.
Figure 2.9: A-B Certain single-qubit errors before a CNOT gate are equivalent to two single-qubit errors happening after the CNOT gate.

[54]. There are alternative approaches such as guage fixing and code deformation [55, 56] but these have their own difficulties.
Chapter 3

Circuit Quantum Electrodymanics

Electrical currents in condensed matter are carried by electrons each following the laws of quantum mechanics. The strong Coulomb force ensures a smooth density of electrons without fluctuations throughout the material\(^1\). Remarkably, this allows us to describe a current not as an enormous number of individual electrons but as an ensemble of electrons with only a few degrees of freedom. In superconductors, we can further ignore low-energy single-particle excitation as these excitations are gapped. The only low-energy excitations left are divergenceless excitations in the ensemble density with charge build-up at boundaries of the material (capacitors). At low temperatures, the low-energy degrees of freedom in the ensemble of electrons behave quantum mechanically with a quantized energy spectrum. As we will see superconducting circuits can form ensemble modes behaving like artificial atoms or harmonic oscillators. Circuit quantum electrodynamics (cQED) is the quantum mechanical description of such coupled atom and oscillator modes \([57]\) analogous to light-matter interactions in cavity quantum electrodynamics, where an atom is coupled to light in a cavity.

This chapter will introduce the ideas of quantized electrical circuits, cQED, and superconducting qubits loosely following notes by Steven M. Girvin \([58]\). The first section describes a quantized Harmonic oscillator both as a lumped element resonator and a distributed circuit. The second section introduces the ideas of Josephson junctions and artificial atoms both with traditional aluminium tunnel junctions and hybrid semiconductor-superconductor junctions. Following the introduction of artificial atoms, the third section describes the interaction of artificial atoms in the resonant and dispersive regimes as well as introducing qubit readout. Last two sections describe qubit control for single-qubit gates and two-qubit coupling and gates for quantum computing algorithms.

\(^1\) Assuming no excitations at frequencies above the plasma frequency of the material. Above this frequency, excitations can form waves in the electron density.
3.1 Quantized Harmonic Oscillators

The simplest harmonic oscillator in an electrical circuit is the LC oscillator in Figure 3.1A. To find the equations of motion of the circuit we first define the node flux at point $\phi$ as our coordinate. A node is a connecting branch between two or more lumped elements [59]. Each node has a node flux defined as

$$\phi(t) = \int_{t'}^t V(t')dt',$$  \hspace{1cm} (3.1)

$$\dot{\phi}(t) = V(t),$$  \hspace{1cm} (3.2)

where $V(t)$ is the voltage at the node. In Figure 3.1A there are two nodes: the upper node defined as $\phi$ and the bottom node defined as ground which by definition has node flux $\phi_{\text{Ground}} = \int_{t'}^t V_{\text{Ground}}(t')dt' = 0$.

The voltage across the inductor can be related to the node flux as

$$\dot{\phi}(t) - \dot{\phi}_{\text{Ground}}(t) = \dot{\phi}(t) = V(t) = LI(t).$$  \hspace{1cm} (3.3)

By integration, we can identify $\phi = LI$ as the magnetic flux stored in the inductor. The energy of the inductor $E_L = LI^2/2 = \phi^2/2L$ in coordinates of $\phi$ looks like a potential energy. Similarly, the energy of the capacitor as a function of $\phi$ is $E_C = CV^2/2 = C\dot{\phi}^2/2$ looks like a kinetic energy. With the potential and kinetic energy of the system we can write the Lagrangian of the system with the node flux $\phi$ as the coordinate:

$$L = \frac{1}{2}C\dot{\phi}^2 - \frac{1}{2}L\phi^2.$$  \hspace{1cm} (3.4)

From the Lagrangian, we identify the conjugate momentum of the node flux $Q = dL/d\dot{\phi} = C\dot{\phi} = CV$ as the charge stored on the capacitor. The Hamiltonian of the system can be found from the Lagrangian with a Legendre transformation

$$H = Q\dot{\phi} - L = \frac{1}{2C}Q^2 + \frac{1}{2L}\phi^2.$$  \hspace{1cm} (3.5)

We recognize the Hamiltonian as that of a harmonic oscillator formed by a particle on

---

2The LC circuit is more commonly solved with the charge of the capacitor as the coordinate. However, when working with Josephson junctions, the node flux is a more convenient choice of coordinate.
a spring, where the particle has coordinate \( \phi(t) \), momentum \( Q(t) \), and mass \( C \) and the spring has spring constant \( 1/L \). With this in mind, the resonance frequency of the harmonic oscillator is readily found as \( \omega = 1/\sqrt{LC} \).

The LC circuit is quantized by promoting the coordinate and its conjugate momentum to quantum operators obeying the canonical commutation relation

\[
[\hat{\phi}, \hat{Q}] = i\hbar.
\]  

The Hamiltonian of the harmonic oscillator can, as usual, be rewritten with raising and lowering operators

\[
\hat{H} = \frac{1}{2C} \hat{Q}^2 + \frac{1}{2L} \hat{\phi}^2 = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right),
\]  

where the raising and lowering operators \( \hat{a}^\dagger \) and \( \hat{a} \) are given by

\[
\hat{a} = \frac{1}{\sqrt{2L\hbar\omega}} \hat{\phi} + i \frac{1}{\sqrt{2C\hbar\omega}} \hat{Q},
\]

\[
\hat{a}^\dagger = \frac{1}{\sqrt{2L\hbar\omega}} \hat{\phi} - i \frac{1}{\sqrt{2C\hbar\omega}} \hat{Q}.
\]

The energy spectrum of the harmonic oscillator is shown in Figure 3.1B with well-known, equidistant energy levels. An eigenstate \( |n\rangle \) of the quantized LC circuit is commonly referred to as a photon number state with \( n \) photons, where \( n \) is the eigenvalue of the number operator \( \hat{n} = \hat{a}^\dagger \hat{a} \). The name originates from light cavities, which are harmonic oscillators whose eigenstates are given by the number of light photons.

Harmonic oscillators formed by lumped element components are instructive to solve to introduce the theory of cQED. However, in practice harmonic oscillators, also commonly referred to as resonators, are often (and exclusively in the work presented in this thesis) formed in distributed elements such as coplanar waveguides (CPWs) shown in Figure 3.2. Distributed CPWs can be modeled as a circuit with inductance \( l \) and capacitance \( c \) per
unit length with a continuous, spatially dependent node flux, \( \hat{\phi}(x,t) \). Microwave cavities are created by introducing boundary conditions such as breaks or shorts of the center conductor in a length of CPW. We will not go through a full derivation of the modes of a distributed cavity, which can be found in [58], and instead focus on the results. The system can be modeled as a sum of non-interacting harmonic oscillators

\[
\hat{H} = \sum_n \left( \hbar \omega_n a_n^\dagger a_n + \frac{1}{2} \right),
\]

where \( \omega_n \) are resonance frequencies described by standing-wave solutions in the spatial degree of freedom of the node flux. For a CPW with wave velocity \( v_p = 1/\sqrt{\epsilon c} \) and wavelength of standing waves \( \lambda_n \) the frequencies are given by \( \omega_n = v_p/\lambda_n \). The wavelengths, \( \lambda_n \), of a cavity depends on boundary conditions of the system. A break in the center conductor as in Figure 3.2 forms a current node (no current can run out of the conductor) and correspondingly a voltage anti-node. Two breaks separated by a length \( L \) creates standing waves with wavelength \( \lambda_n = 2L/n \) with \( n \geq 1 \) each describing a harmonic oscillator mode with resonance frequency \( \omega_n = nv_p/2L \). The voltage oscillation of mode \( n = 2 \) is depicted in pink in Figure 3.2. Such a cavity is known as a \( \lambda/2 \) cavity as its length is half of the wavelength of the lowest mode. If one side instead has a short from center conductor to ground one forms a voltage node as a boundary condition on this side. This cavity will have standing waves with wavelength \( \lambda_n = 4L/(2n + 1) \) with \( n \geq 0 \) and is correspondingly named a \( \lambda/4 \) cavity as \( \lambda_0/4 = L \).

As the resonance frequency of the second-lowest harmonic mode of a distributed cavity is two or three times larger than the lowest mode, one can, in most cases, model it as a single harmonic oscillator described by the lowest frequency mode. For the remainder of this thesis, we will treat distributed cavities as a single harmonic oscillator.

3.2 Artificial Atoms in Superconducting Circuits

As we ultimately are looking to create qubits in superconducting circuits, we need a way to isolate a single two-level system. The energy spectrum of a harmonic oscillator is described by equidistant, non-degenerate energy levels with a single resonance frequency making it impossible to energetically isolate two eigenstates as a qubit. In contrast, the spectrum of an atom is uneven and can have degenerate levels that can readily be utilized as qubits in ion traps. Superconducting artificial atoms are circuits that similarly have uneven energy spectra allowing a qubit subspace to be energetically separated from the rest of the Hilbert space. An uneven energy spectrum is achieved by adding a non-linear element to the circuit\(^3\).

In superconducting circuits, the non-linearity is found as the Josephson effect, which was theoretically predicted by B. D. Josephson in 1962 [62]. Superconductivity originates from an electron-electron interaction that causes electrons to pair up as bosonic Cooper pairs which condense into a boson condensate described by a single wave function \( \psi \)

\(^3\)It is possible to use cavities as qubits by instead implementing nonlinearity in the control circuit [60, 61]. Recent results have shown active error correction in such systems [16].
The magnitude of the wavefunction $|\psi|^2$ is equal to the density of Cooper pairs in the superconductor while its phase only manifests itself when coupling two superconductors. Josephson considered the case of a superconductor-insulator-superconductor (SIS) junction as shown in Figure 3.3. The Cooper pairs in each superconducting electrode can tunnel through the thin insulator allowing a current to flow. Josephson made two predictions for such a weak link Josephson junction:

$$I_s = I_c \sin \varphi,$$
$$\frac{d\varphi}{dt} = \frac{2eV}{\hbar},$$

where $I_s$ is a dissipationless supercurrent tunneling through the insulator and $\varphi$ is the phase difference between the two wavefunctions describing each superconductor. Equation (3.10) is the current-phase relation that describes the dissipationless current flowing across a junction as a function of phase difference $\varphi$. The parameter $I_c$ is the critical current of the Josephson junction given by the maximal dissipationless current that can flow across the junction above which the junction will turn resistive. The energy stored in a Josephson junction as a function of $\varphi$ is readily calculated by combining the two equations (3.10, 3.11):

$$E(\varphi) = \int I_s V(t) dt = \frac{h I_c}{2e} \int \sin(\varphi) d\varphi = -E_J \cos \varphi,$$

where $E_J = hI_c/2e$ is the Josephson energy.

Equation (3.11) is very similar to the definition of node flux $\phi$ given in equation (3.1) leading one to similarly consider $\varphi$ as a position coordinate. With $\varphi$ as a coordinate the energy of (3.12) looks like a potential energy similar to that of an inductor. Importantly the potential energy of a Josephson junction is non-linear. A difference between $\phi$ and $\varphi$ not visible in the equations is that $\varphi$ is a periodic coordinate on the range $[-\pi, \pi]$ while $\phi$ can take any real value. However, in the special case where the wavefunctions of the circuit vanishes at $\varphi = \pm \pi$ we find that $\phi \approx \frac{\Phi_0}{2\pi} \varphi = \frac{\Phi_0}{2\pi} \varphi$, where $\Phi_0 = h/2e$ is the superconducting flux quantum.

Although the potential energy of a Josephson junction resembles that of an inductor

---

4Weak link means that each Cooper pair has a low probability for tunneling through the insulator.
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Figure 3.4: A Josephson junction in parallel with a capacitor and a voltage source coupled capacitively to the circuit.

the current flow is radically different. The current across a junction is carried by single Cooper pairs tunneling across the junction. Consequently, a capacitor plate coupled only through Josephson junctions will have a discreet charge given by an integer number of Cooper pairs. The energy states of the system can be described by charge states $|n\rangle$, where $n$ is the number of Cooper pairs on the capacitor (not to be confused with photon number states introduced in the previous section). A circuit of a Josephson junction in parallel with a capacitor and a nearby voltage source $V_g$ is shown in Figure 3.4. Identifying the energy of the capacitor as the kinetic energy and the potential energy given by the Josephson junction, we can write down the Hamiltonian of the system

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi},$$

where $E_C = e^2/(2(C + C_g))$ is the charging energy of the island, $\hat{n}$ is the number operator for the number of Cooper pairs on the island, and $n_g = -C_gV_g/2e$ is a charge offset. This is known as the Cooper pair box Hamiltonian due to the upper part of the capacitor acting as a box with a discreet number of Cooper pairs. The voltage source $V_g$ describes both the coupling of a controlled charge offset and an uncontrolled environment.

The Cooper pair box Hamiltonian can be simulated numerically in the charge basis with $\hat{n}|n\rangle = n|n\rangle$ and $\cos\hat{\varphi} = \sum(|n\rangle\langle n| + |n+1\rangle\langle n+1|)$ [58]. In Figure 3.5 the energy levels are plotted as a function of the offset charge for different values of $E_J/E_C$. The left panel shows $E_J = E_C$ which is known as the Cooper pair box regime. In this regime, eigenstates are described by a single number of Copper pairs on the capacitor with energies given by parabolas defined by $E_C$ as a function of offset charge $n_g$ (blue dashed lines). The Josephson junction acts as a coupling term between charge states creating avoided crossings between parabola of charge states. The charge dispersion, the change of energy as a function of offset charge $n_g$, arises due to the discretized charge flow through the Josephson junction. While the Cooper pair box can be used as a qubit [64–66] large charge dispersion is undesirable as any charge noise in the vicinity of the capacitor will induce decoherence.

J. Koch et al. proposed a charge-insensitive regime, the transmon regime, defined by $E_J/E_C \gg 1$ [67]. The charge dispersion of the energy levels flattens exponentially with $\sqrt{E_J/E_C}$ making them insensitive to $n_g$ as shown in the right panel of Figure 3.5. While the Hamiltonian is readily solved numerically, it is beneficial to calculate an approximate solution analytically by approximating the Hamiltonian with that of an
Figure 3.5: The lowest energy levels of the Cooper-pair-box Hamiltonian in Equation (3.13) for different values of $E_J/E_C$. The energy of the Hamiltonian with $E_J = 0$ is plotted as light blue dotted parabolas in left panel. In all figures the energy is normalized by $\sqrt{8E_CE_J}$.

LC oscillator. We note that the node flux is proportional to the superconducting phase difference, $\phi = \frac{\Phi_0}{2\pi}\varphi$, and the discreet Cooper pair number $n$ can be related to charge by $Q = 2en$. In coordinates of $\phi$ and $Q$ the Hamiltonian can be written as (setting $n_g = 0$ for the moment)

$$\hat{H} \approx \frac{1}{2C} \hat{Q}^2 - E_J \cos \left(2\pi \frac{\phi}{\Phi_0}\right)$$

$$\approx \frac{1}{2C} \hat{Q}^2 + E_J \left(\frac{2\pi}{\Phi_0}\right)^2 \frac{\hat{\phi}^2}{2}$$

$$= \frac{1}{2C} \hat{Q}^2 + \frac{1}{2L_J} \hat{\phi}^2$$

(3.14)

where we kept only the quadratic term of a Taylor expansion around $\phi = 0$ and $L_J = (\hbar/2e)^2/E_J$ is the inductance of the Josephson junction. The approximate Hamiltonian is that of a Harmonic LC circuit with resonance frequency $\omega = 1/\sqrt{CL_J} = \sqrt{8E_CE_J}/\hbar$.

The Taylor expansion around $\phi = 0$ is only valid if the quantum fluctuations of the solutions are consistent with the assumption $\phi \ll \pi$. The mean square amplitude of the zero point fluctuations is

$$\phi_{ZPF}^2 = \langle 0 | \hat{\phi}^2 | 0 \rangle = \left(\frac{\Phi_0}{2\pi}\right)^2 \left(\frac{2E_C}{E_J}\right)^{1/2},$$

(3.15)

where $|0\rangle$ refers to the ground state of the Harmonic oscillator with raising and lowering operators defined in (3.8). We find that in the transmon limit $E_J/E_C \gg 1$ the Taylor expansion is indeed valid. The same result validates the assumption $\phi = \frac{\Phi_0}{2\pi}\varphi$ as the periodicity of $\varphi$ has no effect for $|\varphi| \ll \pi$.

To second order in the Taylor expansion, the transmon acts as a harmonic oscillator. To show that the transmon is, in fact, an artificial atom with an uneven energy spectrum
the fourth order term of the Taylor expansion is added as a perturbation

\[ \hat{H} \approx \hat{H}_0 + \hat{V}, \]

\[ \hat{V} = -E_J \left( \frac{2\pi}{\Phi_0} \right)^4 \frac{\hat{\phi}^4}{24}, \]

where \( \hat{H}_0 \) is the harmonic Hamiltonian given in Equation (3.14). Using raising and lowering operators of \( \hat{H}_0 \) given in Equation (3.8) we can write \( \hat{\phi}^4 = \left( \Phi_0 / 2\pi \right)^4 \left( \hat{a} \hat{a}^\dagger + \hat{a}^\dagger \hat{a} \right)^4 \).

Inserting into \( \hat{V} \) and dropping all terms with uneven numbers of raising and lowering operators (first-order perturbation theory) the perturbation can be written as

\[ \hat{V} = -\frac{1}{12} E_C \left( \hat{a} \hat{a}^\dagger + \hat{a}^\dagger \hat{a} \right)^4 \approx -\frac{E_C}{2} \left( \hat{a}^\dagger \hat{a} \hat{a} \hat{a}^\dagger + 2 \hat{a}^\dagger \hat{a} \right). \]

In first order perturbation theory this leads to a correction of the energy of state \(|1\rangle\) so that \( E_1 - E_0 = E_{10} = \sqrt{8E_J E_C} - E_C \). For the second excited state \(|2\rangle\) the correction is \(-3E_C\) leading to an energy difference between first and second excited states given by \( E_{12} = \sqrt{8E_J E_C} - 2E_C \). These energy corrections originate from the non-linearity of the cosine potential of a Josephson junction. The amount of non-linearity is quantified by the anharmonicity \( \alpha \) defined by

\[ \alpha = E_{21} - E_{10} \approx -E_C. \]

Remarkably, even the simplest circuit with a Josephson junction leads to artificial atoms with distinct energy spectra depending on the ratio of \( E_J / E_C \). Experimentally, the transmon limit turned out to have longer coherence times due to the suppression of charge noise [68]. However, this comes at the cost of lower anharmonicity, which limits the speed of operations [69], but with optimization of room temperature control equipment [70, 71] this is a much easier problem to work with than inherent charge noise.

### 3.3 Semiconductor-Based Josephson Junctions

Above we described an artificial atom made of a single Josephson junction in the weak coupling regime. Such Josephson junctions are commonly realized by an Al/Al\(_2\)O\(_3\)/Al sandwich with an aluminum oxide thickness of a few nanometers. When fabricated it has fixed characteristics allowing no direct control of the Josephson energy. To gain control of the effective Josephson energy one can place two junctions in parallel to form a SQUID, which has an effective Josephson energy tunable by a magnetic flux. A different approach has become possible as developments in semiconductor growth technology have produced new materials bringing field effect tunability of semiconductors into superconducting circuits [31, 72].

A schematic of a superconductor-semiconductor-superconductor (SSmS) Josephson junction is shown in Figure 3.6. The carrier density of the semiconductor is tunable using a nearby gate which in turn tunes the critical current of the junction. By exchanging the SIS Josephson junction in the transmon circuit with an SSmS junction the transmon
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Figure 3.6: **A** Two superconducting electrodes (blue) sandwiching a semiconductor (green) form a Josephson junction. The semiconductor is tuned by a nearby gate electrode making the Josephson junction gate tunable. **B** Circuit symbol of a gate-tunable Josephson junction.

becomes gate tunable [73–77]. The energy of the gate-tunable transmon ("gatemon") is tuned through the critical current $E_{01}(V_G) \propto \sqrt{E_J(V_G)} \propto \sqrt{I_c(V_G)}$.

Experiments have shown that it is possible to make high-quality semiconductor nanowire proximitized by a superconductor [31, 78]. P. Krogstrup et al. have grown superconducting nanowires with a semiconducting InAs core and an epitaxial aluminium shell, see Figure 3.7A. The perfect crystalline interface between the semiconductor and superconductor makes these nanowires ideal for the development of semiconductor-based superconducting qubits.5 A weak link in the superconducting nanowire is created by chemically etching a small segment of the aluminium shell as shown in Figure 3.7A. The exposed semiconducting InAs core allows electric fields from a nearby gate electrode with voltage $V_G$ to tune the conductance of the core which influences the critical current of the junction. Experimental measurements in Figure 3.7B reveal that the critical current is indeed gate tunable. The critical current is measured as the highest dissipationless current through the junction. The electron mean free path of InAs nanowires has been found to be $l = 100$-150 nm [78, 79]. As the junction length is longer than the mean free path, mesoscopic conductance fluctuations due to scattering across the junction show up as a non-monotonic critical current as a function of gate voltage.

A gate-tunable superconducting artificial atom formed by a nanowire Josephson junction will have different characteristics than that of a conventional transmon [80]. Nanowire-based Josephson junctions have a few highly transmitting channels while the current-phase relation in Equation (3.10) describes the case of many low-transmitting channels. It can be shown theoretically that the potential energy of a ballistic junction with coherence length much longer than the junction width is given by [81]:

$$E = -\Delta \sum_i \sqrt{1 - \tau_i \sin^2(\varphi/2)}, \quad (3.19)$$

where $\tau_i$ is the transmission of the $i$'th channel and $\Delta$ is the superconducting gap. The effective coherence length in an InAs nanowire Josephson junction can be estimated from the superconducting coherence length $\xi_0 \sim 1100$ nm [80] and the mean free path in InAs as $\xi = \sqrt{\xi_0 l} = 300$-400 nm. A typical junction width of $\sim 200$ nm is not much shorter than the coherence length leading to more complicated energy-phase relations [82]. However, experiments have found good agreement with theory for the short junction limit [83–85].

5A weak coupling might create many quasiparticles in the superconductor, which would be detrimental for superconducting qubits.
Figure 3.7: A The nanowire Josephson junction is formed by etching a small segment of the aluminum shell away. A nearby gate electrode tune the conductance of the semi-conducting core. Inset shows the perfect crystalline interface between the InAs core and aluminum shell. B 4-probe resistance measurements of a nanowire-based Josephson junction as a function of gate voltage and current bias. The critical current, $I_c$, of the junction is the lowest current value with non-zero resistance. The extracted critical current is indicated by a blue line.

so this assumption will be taken throughout the thesis.

In the extreme case of unity transmission across the junction, the charge dispersion will completely vanish [86]. While this effect is small away from unity transmission the shape of the potential energy of the Josephson junction additionally modifies the anharmonicity of the gatemon. Following the same procedure as before, but with the energy-phase relation given by Equation (3.19), we can Taylor expand the potential and find the anharmonicity of the artificial atom. To fourth order in $\phi$ the potential is given by [80]

$$E \approx -E_J \left( \frac{2\pi}{\Phi_0} \right)^2 \frac{\hat{\phi}^2}{2} - E_J \left( 1 - \frac{3}{4} \sum \frac{\tau_i^2}{\tau_i} \right) \left( \frac{2\pi}{\Phi_0} \right)^4 \frac{\hat{\phi}^4}{24}.$$  \hspace{1cm} (3.20)

where $E_J = \Delta \sum \tau_i/4$. Here $E_J$ is defined such that the quadratic part has the same form as the transmon leading a harmonic energy spectrum with $\hbar\omega = \sqrt{8E_JE_C}$. The fourth order term can again be written using raising and lowering operators of the unperturbed system:

$$\hat{V} = -E_J \left( 1 - \frac{3}{4} \sum \frac{\tau_i^2}{\tau_i} \right) \left( \frac{2\pi}{\Phi_0} \right)^4 \frac{\hat{\phi}^4}{24} = -E_C \left( 1 - \frac{3}{4} \sum \frac{\tau_i^2}{\tau_i} \right) \left( \hat{a}^{\dagger} + \hat{a} \right)^4 \approx -E_C \left( 1 - \frac{3}{4} \sum \frac{\tau_i^2}{\tau_i} \right) \left( \hat{a}^{\dagger} \hat{a}^{\dagger} \hat{a} \hat{a} + 2\hat{a}^{\dagger} \hat{a} \right).$$  \hspace{1cm} (3.21)

Calculating the first order energy corrections to the eigenstates yields an anharmonicity given by

$$\alpha = -E_C \left( 1 - \frac{3}{4} \sum \frac{\tau_i^2}{\tau_i} \right).$$  \hspace{1cm} (3.22)

High transmission junction lowers the anharmonicity by a factor in between 1 and 1/4. Figure 3.8 shows the potentials generated by low transmission junction, a junction with
unity transmission, and a harmonic potential all with the same harmonic approximation. Indeed we see that the unity-transmission potential more closely resembles the harmonic potential leading to a lower anharmonicity.

### 3.4 Coupled Artificial Atoms and Harmonic Oscillators

With superconducting circuits acting as qubit we need a way to readout the state of the qubit without introducing noise. This can be done by coupling a qubit to a harmonic oscillator, which acts as a filter protecting the qubit from the environment while allowing state readout [57, 87, 88]. Artificial atoms and harmonic oscillators can be coupled through a capacitor $C_g$ as shown in Figure 3.9. Here the resonator is modeled as a lumped-element, LC circuit but the theory also applies to resonators formed in distributed elements as shown in Figure 3.2. The qubit will have a separate coupling to each mode of the distributed cavity, but only one modes has a significant coupling due to the energy separation.

The coupled circuit in Figure 3.9 has three flux nodes: $\phi_A$, $\phi_r$, and ground. Following the same procedure as in section 3.1 the Lagrangian is found as

$$\mathcal{L} = E(\phi_A) - \frac{C_0 \dot{\phi}_A^2}{2} + \frac{\phi_r^2}{2L_r} - \frac{C_g (\dot{\phi}_r - \dot{\phi}_A)^2}{2},$$

(3.23)

where $E(\phi_A)$ is the potential energy of the Josephson junction. Assuming $C_g \ll C_r, C$
the conjugate momenta of each coordinate can be written as

\[
Q_A = \frac{\partial L}{\partial \dot{\phi}_A} \approx C \dot{\phi}_A, \quad (3.24)
\]

\[
Q_r = \frac{\partial L}{\partial \dot{\phi}_r} \approx C_r \dot{\phi}_r. \quad (3.25)
\]

Performing a Legendre transformation and promoting the coordinates and their conjugate momenta to quantum operators yields the Hamiltonian of the system

\[
\hat{H} = \frac{1}{2C} \hat{Q}_A^2 + E(\hat{\phi}_A) + \frac{1}{2C_r} \hat{Q}_r^2 + \frac{1}{2L} \dot{\phi}_r^2 + \frac{C_g}{C C_r} \hat{Q}_r \hat{Q}_A,
\]

\[
= \hat{H}_A + \hat{H}_r + \hat{H}_g \quad (3.26)
\]

where \( \hat{H}_A \) and \( \hat{H}_r \) are the Hamiltonians for the isolated atom and resonator circuits respectively, and \( \hat{H}_g \) is the coupling term. It is convenient to describe the system using eigenstates of the Hamiltonian with \( \hat{H}_g = 0 \). In this case, the eigenstates are simply product states of the uncoupled qubit and resonator which can be described by raising and lowering operators.

Focusing on the low-energy part of the atom spectrum, we find an effective two-level, qubit system. Using raising and lowering operators and Pauli operators, given by \( \hat{\sigma}_z = |1\rangle \langle 1| - |0\rangle \langle 0| \), for isolated resonator and qubit respectively the Hamiltonian becomes\(^7\)

\[
\hat{H} = \hbar \omega_r \hat{a}^\dagger \hat{a} + \frac{\hbar \omega_A}{2} \hat{\sigma}_z + \frac{2eC_2}{C} (1|\hat{n}|0) V_{ZPF}(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-), \quad (3.27)
\]

where \( V_{ZPF} = Q_{ZPF}/C_r = \sqrt{\hbar \omega_r / 2C_r} \) are the voltage zero-point fluctuations of the resonator\(^8\), \( \hat{n} = Q_A/2e \) is the number of Cooper pairs on the capacitor \( C \), \( \hat{\sigma}_+ = |1\rangle \langle 0| \),

---

\(^6\)One finds a small modification to the effective capacitances of the resonator and artificial atom without this approximation as calculated in the appendix of [58].

\(^7\)We have changed the phase of the resonators raising and lowering operators \( \hat{a} \) and \( \hat{a}^\dagger \) as is conventional [58].

\(^8\)For resonators formed in distributed elements \( V_{ZPF} \) is position dependent along the resonator and one needs to calculate \( V_{ZPF}(x) \) at the qubit position \( x \).
The energy spectrum of the Jaynes-Cummings Hamiltonian in the resonant regime with $\omega_r = \omega_q$. On the left are the states $|n, g\rangle$ where $n$ is the number of photons in the resonator and $|g\rangle$ is the ground state of the qubit. Adding a photon to the resonator states increases the energy by $\hbar \omega_r$. On the right are states $|n, e\rangle$ where $|e\rangle$ is the excited state of the qubit raising the energy by $\hbar \omega_q$. In blue are the eigenstates of the coupled system described by Equation (3.29).

$\hat{\sigma}_- = |0\rangle\langle 1|$, and $|i\rangle$ are qubit states. The interaction term can be separated in two terms:

$$h g (\hat{a} + \hat{a}^\dagger) (\hat{\sigma}_+ + \hat{\sigma}_-) = h g (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-) + h g (\hat{a} \hat{\sigma}_- + \hat{a}^\dagger \hat{\sigma}_+),$$

(3.28)

where $h g = \frac{2eC_s}{\epsilon_0} (1|\hat{n}|0) V_{ZF}$. For common experimental parameters, $\omega_q - \omega_r \ll \omega_q + \omega_r$, the second term is negligible due to the large energy difference between the coupled states. Formally, the terms can be neglected with a rotating-wave approximation (in the interaction picture these terms correspond to fast rotating terms averaging to zero) valid when $\omega_q - \omega_r \ll \omega_q + \omega_r$ and $\omega_q, \omega_r \gg g$. Keeping only the first term in the original Hamiltonian, it reduces to the Jaynes-Cummings Hamiltonian:

$$\hat{H} = \hbar \omega_r \hat{a}^\dagger \hat{a} + \hbar \frac{\omega_q}{2} \hat{\sigma}_z + h g (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-).$$

(3.29)

There are two distinct regimes for the Jaynes-Cummings Hamiltonian. The resonant regime when $\omega_r = \omega_q$ and the dispersive regime with $|\omega_q - \omega_r| \gg g$. In the resonant regime, the qubit and resonator states hybridize as shown in Figure 3.10. In the single-excitation manifold, the eigenstates are superpositions of a photon in the resonator and an excitation in the qubit. The splitting of the resonator state is known as the vacuum-Rabi splitting as a qubit excitation does Rabi oscillations with the vacuum state of the resonator. To observe the splitting, we need $g/\pi$ to be larger than the linewidth of both the qubit and the resonator. Observing vacuum-Rabi splitting demonstrates strong and coherent qubit-resonator coupling, but the regime is not suitable for quantum processing.

For quantum processing, we want to be in the dispersive regime where the qubit frequency is far detuned from the resonator frequency. This regime allows us to simplify...
the Jaynes-Cummings Hamiltonian in Equation (3.29) by expanding to second order in the small parameter $g/\Delta$, where $\Delta = \omega_q - \omega_r$ is the detuning. One has to be careful when doing the expansion as higher energy levels of the artificial atom are important. Therefore the expansion is done on the full multilevel system and then truncated to a two-level system afterward [67]. The total system of a multilevel artificial atom coupled to a harmonic oscillator is described by the generalized Jaynes-Cummings Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{V},$$

$$\hat{H}_0 = \hbar \omega_r \hat{a}^\dagger \hat{a} + \hbar \sum \omega_i \langle i | i \rangle,$$

$$\hat{V} = \hbar \sum g_{i,i+1} (\hat{a} | i + 1 \rangle \langle i | + \hat{a}^\dagger | i \rangle \langle i + 1 |).$$

The coupling strength is given by $g_{ij} = \frac{2eC_s}{\hbar C} \langle i | \hat{n} | j \rangle V_{ZPF}$. Here we assume that the artificial atom is a transmon with $E_J \gg E_C$ which leads to $g_{ij} = 0$ for $i \neq i \pm 1$. For other artificial atoms the matrix elements $\langle i | \hat{n} | j \rangle$ can have very different selection rules.

The Hamiltonian can be simplified using second-order perturbation theory treating the interaction term $\hat{V}$ as a perturbation. Eigenstates for $H_0$ are readily found as $| n, j \rangle$ where $n$ is the resonator photon number and $j$ is the excitation level of the artificial atom. An explicit calculation can be found in Appendix A leading to the Hamiltonian:

$$\hat{H} = \hbar \left( \omega_r - \frac{\chi_{12}}{2} \right) \hat{a}^\dagger \hat{a} + \frac{1}{2} (\omega_q + \chi_{01}) \hat{\sigma}_z + \hbar \chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z$$

(3.31)

where $\chi_{ij} = g^2_{ij} / (\omega_{ij} - \omega_r)$ and $\chi = \chi_{01} - \chi_{12}/2$. Figure 3.11 depicts the lowest energy levels of the Jaynes-Cummings Hamiltonian in the dispersive regime.

There are three terms in the Hamiltonian originating from the coupling. The first two terms are called Lamb shifts giving a correction to the qubit and resonator frequencies. The last term can be interpreted in two ways. It can be viewed as a correction to the qubit frequency dependent on the number of photons in the resonator [89]. This is known as the Stark shift of the qubit and can be exploited to measure photon number states in the resonator [60, 90]. Equally valid it can be interpreted as a qubit dependent dispersive shift of the resonator:

$$\hat{H} = \hbar \left( \omega_q' + \chi \sigma_z \right) \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega_r' \hat{\sigma}_z,$$

(3.32)

where $\omega_q' = \omega_q + \chi_{01}$ and $\omega_r' = \omega_r - \chi_{12}$. Written in this form, the Hamiltonian explicitly shows a qubit state dependent shift on the resonance frequency of the resonator. The dispersive shift in the transmon limit is given by $\chi = \alpha g^2 / \Delta(\Delta + \alpha)$, where $\alpha$ is the anharmonicity of the qubit. By probing the frequency of the resonator with a microwave signal, we can infer the qubit state. Furthermore, this is a quantum non-demolition (QND) readout scheme as the qubit state is an eigenstate of the Hamiltonian, which means that the qubit is left in the measured state after readout [57]. This can be exploited to perform qubit state preparation with fast measurement feedback [91, 92].

The resonator is coupled to the measurement apparatus leading to a photon decay.
rate, $\kappa$, or the resonator. As the qubit is coupled to the resonator, the photon decay will induce a qubit decay known as the Purcell effect [93]. For large detuning the induced qubit decay is given by $\gamma \approx (g/\Delta)^2 \kappa$ [88]. The speed of qubit readout is limited by $\kappa$, the rate of photons leaking out to instruments, while qubit lifetime is limited by $1/\kappa$. Depending on the scope of the experiment one might need to suppress the Purcell effect to allow for fast measurements without compromising qubit lifetimes [94–97].

### 3.5 Single Qubit Control

For qubit control, we return to the simple transmon circuit capacitively coupled to a voltage source [Figure 3.4] whose Hamiltonian is

$$\hat{H} = 4E_C \hat{n}^2 - E_J \cos(\hat{\phi}) + \frac{2eC_2}{C^2} V_g(t) \hat{n},$$

(3.33)

where $V_g(t)$ is separated as an individual term. Qubit operations are achieved by applying an ac voltage $V_g(t) = v_R \cos(\omega t) + v_I \sin(\omega t)$ where $v_R$ and $v_I$ are the in phase and out of phase components of the voltage respectively. Writing the Hamiltonian in eigenstates of the undriven artificial atom we have

$$\hat{H} = \sum_i \hbar \omega_i |i\rangle \langle i| + \sum_{i,j} 2e\beta \langle i| \hat{n} |j\rangle \left[ v_R \cos(\omega t) + v_I \sin(\omega t) \right] (|j\rangle \langle i| + |i\rangle \langle j|),$$

(3.34)
where $\beta = C_g/C$. Focusing on a two-level subspace spanned by states $|0\rangle$ and $|i\rangle$ the Hamiltonian can be written with Pauli operators:

$$
\hat{H} = \frac{\hbar \omega_i}{2} \hat{\sigma}_{z,i} + 2 e \beta \langle 0 | \hat{\sigma}_{z,i} | i \rangle [v_R \cos(\omega t) + v_I \sin(\omega t)] (\hat{\sigma}_{+,i} + \hat{\sigma}_{-,i}),
$$

(3.35)

where $\hat{\sigma}_{z,i} = |i\rangle\langle i| - |0\rangle\langle 0|$, $\hat{\sigma}_{+,i} = |i\rangle\langle 0|$, and $\hat{\sigma}_{-,i} = |0\rangle\langle i|$. In a rotating frame of the drive and invoking the rotating-wave approximation the Hamiltonian reduces to

$$
\hat{H}_R = e^{i\omega t \hat{\sigma}_{z,i}/2} \hat{H} e^{-i\omega t \hat{\sigma}_{z,i}/2} = \frac{\hbar}{2} (\Omega_{R,i} \hat{\sigma}_{x,i} - \Omega_{I,i} \hat{\sigma}_{y,i}),
$$

(3.36)

where $\Omega_{j,i} = \frac{2}{\pi} \beta \langle 0 | \hat{n} | i \rangle v_j$ are Rabi frequencies. A classical microwave signal $V(t)$ on an electrode capacitively coupled to the artificial atom can drive the system from $|0\rangle$ to $|i\rangle$ and back with a frequency given by $\Omega_{j,i}$. With independent control of $\Omega_{R,i}$ and $\Omega_{I,i}$ we can drive the two-level system around an arbitrary axis in the XY plane of the Bloch sphere. For transmons with $E_J \gg E_C$ the only non-zero matrix elements are $\langle i | \hat{n} | i + 1 \rangle$ allowing us to focus on just the 0-1 transition. However, as we will see in Chapter 6, more exotic circuits can have tunable matrix elements leading to some transitions appearing and disappearing as they are tuned.

For tunable transmons, e.g. gatemons, one can tune the qubit frequency. Limiting the Hamiltonian to a truncated qubit subspace with resonance frequency $\omega_q$ the Hamiltonian of a gatemon can, in the rotating frame of the drive, be written as

$$
\hat{H}_R = \frac{\hbar}{2} [\delta_q(V_c) \hat{\sigma}_z + \Omega_{R} \hat{\sigma}_x - \Omega_{I} \hat{\sigma}_y],
$$

(3.37)

where $\delta_q(V_c) = \omega_q(V_c) - \omega$ is the qubit-drive detuning and $V_c$ is the control voltage tuning $E_J$ of the Josephson junction. With independent and fast control of all parameters $\delta_q$, $\Omega_{R}$, and $\Omega_{I}$ we have complete control of the qubit system.

Here we considered a drive signal, $V_g(t)$, applied to a nearby electrode capacitively coupled to the artificial atom. Alternatively, one might apply the drive signal through a readout cavity coupled to the artificial atom. In this case the cavity will act as a filter on the drive signal reducing the effective Rabi frequencies dependent on the cavity-qubit coupling and detuning: $\Omega_j = \frac{\hbar}{\delta} \left[ \frac{2}{\pi} \beta \langle 0 | \hat{n} | i \rangle v_j \right]$, but otherwise behaves the same as a direct capacitive coupling [58].

### 3.6 Two-Qubit Operations

For universal quantum processing, we also need to engineer qubit-qubit interactions. Fortunately, it is sufficient to have just one entangling two-qubit gate. For transmon qubits there are several ways to implement two-qubit gates [98–101]. Here we will focus on one of the most widely used two-qubit gates: the controlled phase gate (CZ gate) [102, 103]. The CZ gate performs a $Z$ gate on a target qubit dependent on the state of a control qubit. Implementations of two-qubit gates rely both on an engineered coupling
CHAPTER 3. CIRCUIT QUANTUM ELECTRODYNAMICS

Figure 3.12: Two transmon qubits coupled capacitively.

and control pulses used to perform the gate.

There are multiple ways to engineer qubit-qubit couplings for transmons. One is a direct capacitive coupling that is very similar to the qubit resonator coupling [104] while another is a coupling mediated by a resonator [105], which was used for the first demonstration of two-qubit operations in transmon qubits.

Two transmons can be coupled capacitively as shown schematically in Figure 3.12. Notice the similarity of the circuit to that of a transmon coupled to a harmonic oscillator in Figure 3.9. Following the same procedure leading to the Jaynes-Cummings Hamiltonian in Equation (3.29) but truncating both transmons to two-level systems it is straightforward to find the Hamiltonian as

\[
\hat{H} = \hbar \frac{\omega_1}{2} \sigma_{z,1} + \hbar \frac{\omega_2}{2} \sigma_{z,1} + \hbar J (\sigma_{-,1} \sigma_{+,2} + \sigma_{+,1} \sigma_{-,2}),
\]

(3.38)

where \( \omega_1 \) and \( \omega_2 \) are the resonance frequencies of qubit 1 and 2 respectively and \( J = \frac{(2e)^2 C_g}{C_1 C_2} \langle 0 \mid \hat{n}_1 \hat{n}_1 \rangle \langle 1 \mid \hat{n}_2 \hat{n}_2 \rangle \) is the qubit-qubit coupling strength. In the transmon limit we can write the coupling term as \( J = \frac{C_g \sqrt{\omega_1 \omega_2}}{2 \sqrt{C_1 C_2}} \). If the qubits are far detuned in frequency, the coupling term becomes negligible due to energy conservation. By pulsing the qubits into resonance, for instance by changing the gate voltage on a nanowire Josephson junction, one can turn on the coupling for a short time to perform a gate.

A somewhat more involved system is the qubit-resonator-qubit circuit shown in Figure 3.13. The Hamiltonian in the rotating wave approximation takes the form of a Jaynes-Cummings Hamiltonian with qubit-resonator couplings for each qubit

\[
\hat{H} = \hbar \omega_r \hat{a}^\dagger \hat{a} + \hbar \sum_i \frac{\omega_i}{2} \sigma_{z,i} + \sum_i \hbar g_i (\hat{a} \sigma_{+,i} + \hat{a}^\dagger \sigma_{-,i}).
\]

(3.39)

This Hamiltonian is known as the Tavis-Cummings Hamiltonian and describes the cou-
pling of multiple qubits to a single harmonic mode. As for readout, we want to be in the
dispersive limit where both qubits are far detuned from the resonator. In the dispersive
limit where $g_1, g_2 \ll \Delta_1, \Delta_2$ the Hamiltonian can be written as [106]

$$\hat{H} = \hbar (\omega'_r + \chi_1 \hat{\sigma}_{z,1} + \chi_2 \hat{\sigma}_{z,2}) \hat{a}^\dagger \hat{a} + \sum_{i=1}^2 \frac{\hbar \omega'_i}{2} \hat{\sigma}_{z,i} + \hbar g_1 g_2 \frac{\Delta_1 + \Delta_2}{2\Delta_1 \Delta_2} (\hat{\sigma}_{+,i} \hat{\sigma}_{-,i} + \hat{\sigma}_{-,i} \hat{\sigma}_{+,i}),$$  
(3.40)

where $g_i = \frac{2eC_i}{\epsilon_D} \langle 1 | \hat{n}_i | 0 \rangle V_{ZPF}$ and $\Delta_i = \omega_i - \omega_r$. The coupling term is the same as for
the direct coupling with a strength determined by the qubits’ coupling to the resonator
and how far detuned they are. When the two qubits are on resonance, the coupling
strength is $g_1 g_2 / \Delta$.

Both implementations of qubit-qubit coupling lead to the same effective Hamiltonian.
Experimentally there are pros and cons of both layouts and the choice of coupling depends
on the specific experiments needs. A distributed coupling cavity allows extra space as
the qubits can be coupled over long distances. This suppresses any unwanted crosstalk
between qubits for single-qubit gates as well as allowing extra space for control wirings
such as readout resonators and gate lines. On the other hand, adding a cavity to mediate
the coupling also adds a decay channel as well as one more element that can fail during
fabrication.

The coupling between qubits in this implementation is a fixed coupling strength which
is dynamically turned on by pulsing the qubits into resonance. Figure 3.14 shows the
level spectrum for two coupled transmons. Blue lines are the single-excitation energies,
with an avoided crossing between states $|01\rangle$ and $|10\rangle$, while red shows two-excitation
energies and coupling between states $|02\rangle$ and $|11\rangle$. Two types of two-qubit gates can be
performed in this spectrum. One is an iSWAP gate performed by pulsing the energy of
qubit 1 diabatically into the $|10\rangle$-|01\rangle anticrossing for a certain time$^{10}$:

$$iSWAP(00) = |00\rangle, \quad iSWAP(11) = |11\rangle,$$
$$iSWAP(01) = -i|10\rangle, \quad iSWAP(10) = -i|01\rangle.$$  
(3.41)

The gate set of iSWAP and single qubit gates is, in fact, a universal gate set [107].

Unfortunately, due to the low anharmonicity of a transmon qubit one has to consider
the effect of higher energy states shown in red. The coupling term in Equation (3.38) also
couples states $|02\rangle \leftrightarrow |11\rangle$ and $|20\rangle \leftrightarrow |11\rangle$. To avoid any leakage errors the pulse scheme
used to implemented two-qubit gates needs to suppress any $|02\rangle$ or $|20\rangle$ population after
each gate. This poses a problem for the iSWAP operation which requires a diabatic pulse
to bring the two qubits on resonance. Such a pulse will have to move through the avoided
crossing between states $|02\rangle$ and $|11\rangle$ (or $|20\rangle$ and $|11\rangle$), which will cause leakage errors
if the pulse is not fast enough. Experimentally it turns out to be challenging to avoid
any leakage leading experimenters to come up with another type of two-qubit gates in
transmons.

$^{10}$Such a pulse will also perform a single qubit phase operation which for simplicity has not been
included in this discussion. One can easily correct for the phase operation, e.g. with a single qubit phase
operation after the two-qubit operation.
The idea is to take advantage of the $|02\rangle \leftrightarrow |11\rangle$ anticrossing while avoiding any leakage by clever pulse shaping [102]. A diabatic pulse into the $|02\rangle \leftrightarrow |11\rangle$ anticrossing will after a time implement a controlled phase (CZ) gate due to state oscillations of $|11\rangle$ and $|02\rangle$:

$$
\begin{align*}
\text{CZ}|00\rangle &= |00\rangle, \\
\text{CZ}|11\rangle &= -|11\rangle, \\
\text{CZ}|01\rangle &= |01\rangle, \\
\text{CZ}|10\rangle &= |10\rangle.
\end{align*}
$$

The CZ gate avoids the complexity of interacting with multiple anticrossings in the same pulse. More recent implementations make use of a fast adiabatic approach to the avoided crossing keeping the state in an eigenstate at all times [103, 108]. Such a pulse has the advantage of limiting leakage simply by making the pulse slower. Furthermore, it is experimentally much easier to implement adiabatic pulses than diabatic due to the limited bandwidth of control electronics and cabling.
Chapter 4

Fabrication and Experimental Setup

4.1 Fabrication

Device fabrication of superconducting qubits is an extended process requiring several advanced techniques. Furthermore, energy losses in superconducting qubits can easily be limited by the materials and the quality of the materials interfaces formed during fabrication [109–112]. Consequently, optimizing qubit lifetimes is a tight loop with the fabrication processes.

For nanowire-based devices the fabrication is further complicated by the somewhat random placement of the nanowire requiring manual design adjustments for each nanowire\(^1\). The individual steps of the process are fairly standard involving either e-beam or UV lithography followed by a lift-off process or an etch-back process. A detailed list of processes for each sample presented in this thesis can be found in Appendix D. The general process flow of nanowire-based superconducting circuits can be divided into two stages: before nanowire deposition and after nanowire deposition.

The first stage defines the large-scale cQED circuits, such as resonators and qubit islands. At this level, every device is identical and can beneficially be fabricated simultaneously on a single wafer. For small-scale cleanrooms, such as the one used at Center for Quantum Devices, 2” wafers containing a few tens of chips is a good compromise between parallel fabrication and tool availability. As this stage is a parallel fabrication of tens of devices, one wants to do as much of the fabrication as possible during this stage.

Before the second stage, the wafer is cut up in smaller chips containing only a few devices. Much of the work in this stage scales with the number nanowires rather than the number of chips. The nanowire placement can be done in two fashions. A random placement via a tissue transferring tens of nanowires from growth chip to device chip as described in [114]. This is a reasonably fast process but is somewhat uncontrolled. Alternatively one can use a micromanipulator, which is a needle controlled to sub-micrometer

\(^1\)This can be mitigated with image recognition software [113].
scale, installed under a microscope to transfer individual nanowires from the growth chip to the desired location and orientation on device chip. This is a very tedious and time-consuming process but necessary if the placement of the nanowire is crucial, e.g. bottom gate structures.

Following placement of InAs/Al core/shell nanowires, a small segment of the shell needs to be etched away to form Josephson junctions. The process resulting in best nanowire etches during this work has been a 9-12 s, 50° C Transcene D etch with e-beam lithography windows defined in PMMA directly followed by thorough rinsing in DI water. While giving good results, it is a very sensitive process with far from 100 % yield.

4.2 Experimental Setup

Superconducting qubit devices are sensitive to radiation both at the qubit frequency and in THz frequencies (infrared light) which generates quasiparticle excitations. Consequently, it is crucial to properly shield devices from any black-body radiation of higher temperature stages [115, 116]. This is commonly done by multiple closed boxes with light absorbing coatings. Any residual light entering a box should be absorbed far away from the device. Similarly, electrical connections to the device are carefully filtered or heavily attenuated. Detailed schematics of each setup used are given in Appendix C.

For measurements in Chapters 5 and 6 we use an aluminium box inside a copper box both coated by light-absorbing paint, Aeroglaze Z306. Figure 4.1A shows the aluminium box and lid with the black, light-absorbing coating. An indium wire, which is exchanged if the box is opened, is placed around the device cavity for a light-tight seal to the lid. The superconducting, aluminium box furthermore suppresses any changes in magnetic field. A copper shim is placed on top of the device to reduce any vacuum cavity modes near the qubit frequency.

The box for high-field compatibility used in Chapter 7 is made of copper beryllium (CuBe) [Figure 4.1B]. The high thermal conductance of CuBe as well as its relatively low conductance, for reduced heating from eddy currents, makes CuBe an excellent material for high-field, low-temperature applications. The light-absorbing paint is replaced by a non-magnetic, light-absorbing foam, Eccosorb LS-26, filling the cavity around the device due to concerns that Aeroglaze Z306 might lose its absorbing properties, which are related to ferromagnetic spins in the paint, at high magnetic fields. To enable tens of DC connections to the device the box has been designed with two chambers: a filtering chamber and a device chamber. The unshielded DC lines enter the filtering chamber where surface mount 80 MHz low-pass filters remove any high-frequency radiation coupling to the lines (not necessary for other cables due to their coaxial nature).

Figure 4.2 shows the typical low-temperature filtering of the readout circuit. The input is heavily attenuated suppressing any room-temperature radiation. A combination of 10 GHz low-pass filters and absorptive, home-made Eccosorb filters ensures no radiation above 10 GHz. The attenuation of the Eccosorb CR-110 filters has a linear

\[ \text{Etch time is dependent on shell thickness: 9 s for 7 nm half-shell and 12 s for 30 nm thick full-shell nanowires.} \]
frequency dependence with $\sim 1$ dB attenuation at 5 GHz. Eccosorb filters were omitted on setups for high magnetic fields as their interaction with magnetic fields is unknown. On the output side, cryogenic isolators suppress noise from the HEMT amplifier without attenuating the signal from the device.

Current and voltage bias lines are designed for a bandwidth up to $\sim 300$ MHz to allow pulses for qubit control with widths of a few tens of nanosecond [Figure 4.3]. Higher frequencies are filtered by a combination of 300 MHz low-pass filters and homemade Eccororb CR-124 filters. Eccosorb CR-124 filters are more attenuating than the ones used for readout circuits with 3 dB attenuation at $\sim 350$ MHz. For voltage bias lines the DC signal cannot be attenuated by standard voltage dividers as these operate with low resistance resistors. Rather the DC signal is separately filtered with QFilters from QDevil (www.QDevil.com) and then combined with the high-frequency part via a bias-T consisting of a 10 k$\Omega$ and a 5.1 nF capacitor.

Figure 4.1: Box types used for device shielding. A Aluminium box used for zero field experiments. B Gold-plated CuBe box used for high field experiments.

Figure 4.2: Input and output lines taking readout signals from room temperature to the sample at $< 50$ mK and back up to room temperature.

Figure 4.3: Bias-T consisting of a 10 k$\Omega$ and a 5.1 nF capacitor.
4.3 Measurement Techniques

Throughout the thesis two main setups techniques have been used: One based around a vector network analyzer (VNA) for measurements of resonance frequencies of readout cavities and two-tone spectroscopy of transition frequencies in artificial atoms. The second is a time-resolved setup controlled by an arbitrary waveform generator (AWG) used to probe time-domain behavior and coherence of artificial atoms. We use a microwave switch to switch between the two independent measurement techniques.

4.3.1 Network Analyzer Measurements

Circuit resonances, such as resonances of readout cavities, are commonly probed with a network analyzer measuring the transmission coefficient of the circuit. A VNA measures both the amplitude and phase response of a circuit as a function of probe frequency while a scalar network analyzer (SNA) measures only the amplitude response. The network analyzer is mainly used to measure the resonance frequencies of the readout cavities for which an SNA is sufficient as shown in Figure 4.4A. From the measurement of the cavity resonance, the transmission of a single frequency near the resonance is chosen to be monitored to infer changes in the qubit state (red point in Figure 4.4A). The specific choice of readout frequency, if it is near the resonance frequency, only weakly affects signal strength of subsequent qubit measurements. However, if long measurements are to be performed at fixed qubit parameters, it is prudent to first optimize the readout frequency by measuring the signal-to-noise ratio as a function of readout frequency.

Two-tone spectroscopy, used to probe resonances of an artificial atom coupled to a readout cavity, is performed by monitoring the transmission at a chosen readout frequency while the frequency of a second microwave tone (the drive tone) is being swept [Figure 4.4B]. This is often performed by stepping the frequency of the microwave signal generator while monitoring the response in transmission. However, signal generators are commonly slow at stepping frequency leading to a great deal of ‘dead time’ waiting for the instrument setting. For faster data acquisition we take advantage of features on a Rhode & Schwarz ZNB VNA allowing to decouple output frequency and measurement frequency (it is possible other brands of VNAs has similar features). A VNA is optimized for frequency sweeping which we utilize for the drive tone in two-tone spectroscopy. Depending on the version of the VNA we use two different setups shown in Figure 4.5 for
Figure 4.4: 

- **A** Transmission of the readout cavity as a function of probe frequency. Red point indicates the chosen readout frequency to be monitored in two-tone spectroscopy.
- **B** A second microwave drive tone is swept while monitoring the transmission of the readout cavity at the red circle in **A**. Two peaks are observed as the drive frequency is on resonance with transition frequencies of the artificial atom.

Two-tone spectroscopy. A 4-port Rhode & Schwarz ZNB VNA, used in Figure 4.5A, has two separate signal generators. One signal generator is set to measure the transmission of the fixed readout frequency while the frequency of the second signal generator is supplying the drive tone being swept. The result is a measurement of the transmission at the readout frequency as a function of the frequency of a drive tone as desired - all performed with a single instrument. The second setup in Figure 4.5B is based around a 2-port Rhode & Schwarz ZNB VNA which only has a single signal generator. The signal generator of the VNA is again utilized as the drive tone with fast frequency sweeping. To monitor the transmission of the readout frequency, an external signal generator is set to output the readout tone supplied to the device while the VNA is configured to monitor the input at the readout frequency at the receiving port. It is crucial to have a common 10 MHz reference between the signal generator and the VNA to avoid differences in the generated and measured frequencies. This setup is inferior to the 4-port version as only amplitude data can be measured due to a missing phase reference between the signal generator and the VNA. Fortunately, by optimizing the choice of readout frequency one can ensure that almost all the signal is contained in the amplitude response. Detailed commands for the R&S ZNB VNA used can be found in the manual under "4.7.3 Frequency Conversion Measurements" and "5.12.2.2 Port Settings Dialog".

As mentioned, two-tone spectroscopy probes the resonances of the artificial atom with the readout cavity acting as a detector. However, in some measurements, both the cavity and the artificial atom are modified. Any changes to the resonance frequency of the readout cavity, for example due to a large push from the artificial atom, will heavily modify the signal in two-tone spectroscopy. This can be circumvented by utilizing adaptive measurements that track the resonance frequency of the readout cavity either by a beforehand measured look-up table or by measuring the cavity resonance before each two-tone spectroscopy measurement.
CHAPTER 4. FABRICATION AND EXPERIMENTAL SETUP

Figure 4.5: A A single 4-port VNA monitors a fixed readout tone output at P1 and received at P2 while a drive tone output at P3 is swept. B A 2-port VNA monitors a fixed readout tone output by an external signal generator and measured at P2 on the VNA while a drive tone output at P1 is swept. Crucially a 10 MHz reference ensures the signal generator and VNA outputs and measures at the exact same frequency.

Figure 4.6: Schematic of a microwave mixer ideally acting as a signal multiplier. The local oscillation (LO) can be multiplied with either (A) the radio frequency (RF) input or (B) the intermediate frequency (IF) input. The three input signals work in two frequency ranges with \( f_{RF} \sim f_{LO} \gg f_{IF} \).

4.3.2 Time-Domain Measurements

The coherence of a qubit state is probed using time-domain measurements where the manipulation of the state is separated from the readout. Each readout will measure the \( Z \)-eigenvalue of the qubit while the average of many measurements, where the qubit is prepared to the same state, will yield the expectation value \( \langle Z \rangle \). The expectation value \( \langle Z \rangle \) can this way be measured as a function of qubit manipulation parameters. Particularly, the approximate time-evolution of the quantum state during the manipulation is measured by varying for how long the manipulation is performed. In practice, this is achieved by combining different instruments and control software to carefully control and measure the quantum state. This subsection will first cover the necessary room temperature circuitry for state manipulation followed by different readout techniques.

In time-domain measurements, microwave signals with frequencies of a few GHz need to be modulated with a time-resolution on the order of nanoseconds. Instruments capable of directly synthesizing signals at such frequencies and modulation are exceedingly expensive. Instead, the microwave signals are generated by microwave signal generators and modulated by AWGs with mixers.

A mixer is a passive device, which acts as a signal multiplier. The three ports are usually named radio frequency (RF), local oscillator (LO), and intermediate frequency (IF) with RF and LO signals at high frequencies (typically a few GHz for cQED) and IF at
lower frequencies ($< 1$ GHz). When applying signal $V_{LO}(t)$ and $V_{IF}(t)$ to the respective ports of the mixer the resulting signal at the RF port will be $V_{RF}(t) = V_{LO}(t)V_{IF}(t)$ [Figure 4.6A]. The mixer also works in reverse when applying signals $V_{LO}(t)$ and $V_{RF}(t)$ resulting in $V_{IF}(t) = V_{LO}(t)V_{RF}(t)$ [Figure 4.6B].

For quantum state manipulation an IQ mixer, which internally consists of two mixers and a 90° hybrid, is utilized for IQ modulation [Figure 4.7]. The 90° hybrid splits the input microwave signal, $V_{LO} = \cos(\omega t)$, in two paths with a 90° phase shift between them. Using two mixers, these signals are multiplied with separate control signals $I(t)$ and $Q(t)$ and combined to a single signal given by

$$V_{RF}(t) = I(t) \cos(\omega t) + Q(t) \sin(\omega t).$$

(4.1)

The resulting signal is exactly of the form required for qubit manipulation as shown in Section 3.5 where $\Omega_R(t) \propto I(t)$ and $\Omega_I(t) \propto -Q(t)$. The two components $I(t)$ and $Q(t)$ are known as the in-phase (or real) and quadrature (or imaginary) components of the signal, hence the name IQ modulation. With an AWG generating signals $I(t)$ and $Q(t)$, qubit rotations are easily controlled simply by uploading different waveforms. Many signal generators have an internal IQ mixer, called vector signal generators or IQ signal generators, to enable IQ modulation without further circuit components.

Microwave signals and IQ modulations enable control of rotations around the $x$ and $y$ axis of the Bloch sphere. For rotations around the $z$ axis as well as qubit-frequency pulsing for 2-qubit operations, baseband pulses are sent directly from the AWG to the nanowire gate modifying $E_J(V_G)$ on nanosecond timescales. While the AWG controls all rotation the microwave excitation signals and voltage pulses have different paths leading to small differences in the time it takes for the signals to reach the sample. The time discrepancy is usually measured by varying the time between a microwave $\pi$ pulse and a $z$ rotation as the $z$ rotation will only affect the measurement outcome if applied simultaneously with the $\pi$ pulse. All subsequent control sequences are then corrected such that microwave pulses and $z$ rotations arrive at the sample as desired.

The above describes the case of ideal circuitry which is sufficient for measuring coherent behavior of qubits. However, for small distortions of the signal due to imperfect circuit components and limited bandwidth will limit the performance of high-fidelity gates. Several techniques have been developed to compensate for circuit imperfections

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
  \centering
  \includegraphics[width=\textwidth]{figure.png}
  \caption{A Schematic of internal components in an IQ mixer.}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
  \centering
  \includegraphics[width=\textwidth]{figure.png}
  \caption{B Circuit symbol of an IQ mixer.}
\end{subfigure}
\caption{A Schematic of internal components in an IQ mixer. B Circuit symbol of an IQ mixer.}
\end{figure}
by utilizing qubit measurements as the optimization parameter [106, 108, 117].

For qubit readout, the same IQ modulation is utilized to turn on the readout signal shortly after the qubit manipulation is done. Similar to the case of two-tone spectroscopy with a VNA, we want to measure the amplitude and phase of the transmitted signal. Figure 4.8 shows the setup for heterodyne readout where the amplitude and phase of a transmitted microwave tone with frequency $f_r$ is measured. The microwave tone, initially at $f_r \sim 5 - 8$ GHz, is first down converted to a lower frequency $f_d \sim 10 - 300$ MHz using a mixer to avoid requiring too high sampling rates on the ADC. The resulting signal measured by the ADC is given by

$$A_r \cos(\omega_d t + \phi_r) \cos(\omega_{LO} t) = A_r A_{LO} \frac{1}{2} \{ \cos[(\omega_r - \omega_{LO}) t + \phi_r] + \cos[(\omega_r + \omega_{LO}) t + \phi_r] \}$$

Low-pass filter $A_r A_{LO} \frac{1}{2} \cos(\omega_d t + \phi_r)$,

(4.2)

where the high-frequency component is suppressed by the low-pass filter and $\omega_d = \omega_r - \omega_{LO}$. The amplitude and phase, $A_r$ and $\phi_r$, the signal is demodulated in software by multiplication with digitally-generated sine and cosine signals (here setting $A_{LO} = 4$ as it acts only as a global scale factor):

$$2A_r \cos(\omega_d t + \phi_r) \cos(\omega_{LO} t) = A_r \cos(\phi_r) + A_r \cos(\omega_d t + \phi_r)$$

$$2A_r \cos(\omega_d t + \phi_r) \sin(\omega_{LO} t) = A_r \sin(\phi_r) + A_r \sin(\omega_d t + \phi_r).$$

(4.3)

A digital low-pass filter removes the high-frequency component leaving the signals $X_d = A_r \cos(\phi_r)$ and $Y_d = A_r \sin(\phi_r)$ from which the amplitude and phase are readily extracted as $A_r = \sqrt{X_d^2 + Y_d^2}$ and $\phi_r \tan^{-1}(Y_d/X_d)$.

In practice the basic setup for heterodyne readout has some limitations as the phase difference between the two signal generators will drift over time even with a 10 MHz reference signal. Usually, the phase drift is circumvented by a reference signal as shown in Figure 4.9A. Without loss of generality, we can assume the full phase drift is happening on the local oscillator signal. The local oscillator signal is then given by $A_{LO} \cos(\omega_{LO} t + \phi_r)$.

Figure 4.8: A microwave tone probing the readout cavity is down converted using a second slightly detuned signal generator. A low pass filter removes any high-frequency components from the down conversion before the signal is digitized by an ADC. Software demodulation extracts the magnitude and phase of the down-converted signal (see main text).
Figure 4.9: Two methods to avoid phase noise between signal generators. 

A A separate reference signal path is created and measured similarly to the main signal. Any phase noise affects both identically and can be subtracted in software.

B Utilizing a feature on R&S SGS100A IQ signal generator the microwave tone is output before IQ modulation at LO Out. With SSB modulation the signal frequency at RF Out is moved by $\omega_d$ allowing for heterodyne demodulation similar to Figure 4.8 with only one signal generator hence eliminating the need for a reference.
\(\phi_{LO}\), where \(\phi_{LO}\) is the uncontrolled phase drift. When included in the calculations above the measured signals are \(X_d = A_r \cos(\phi_r - \phi_{LO})\) and \(Y_d = A_r \sin(\phi_r - \phi_{LO})\). Two reference signals \(X_{\text{ref}} = A_{\text{ref}} \cos(\phi_{\text{ref}} - \phi_{LO})\) and \(X_{\text{ref}} = A_{\text{ref}} \sin(\phi_{\text{ref}} - \phi_{LO})\), where the constants \(A_{\text{ref}}\) and \(\phi_{\text{ref}}\) are amplitude and phase of the reference path, are measured in a similar fashion. The phase drift can then be removed at the cost of a constant phase shift, \(\phi_{\text{ref}}\), given by the length of the reference path. The measured phase of the signal is then given by \(\phi_{r,\text{ref}} = \tan^{-1}(Y_d/X_d) - \tan^{-1}(Y_{\text{ref}}/X_{\text{ref}}) = \phi_r - \phi_{\text{ref}}\).

A more convenient way of removing the phase drift relies on features available on a Rhode & Schwarz SGS100A IQ vector signal generator (other vector signal generators might have similar features). Figure 4.9B displays the setup where the signal generator outputs part of the microwave signal at a second port before being modulated by the IQ mixer. The main signal going to the IQ mixer is moved in frequency by single-sideband (SSB) modulation. SSB modulation is performed by applying \(I(t) = \cos(\omega_d t)\) and \(Q(t) = -\sin(\omega_d t)\), signals readily generated by an AWG, to the IQ mixer with the resulting microwave tone given by

\[
V_{RF}(t) = I(t) \cos(\omega t) + Q(t) \sin(\omega t) = \cos(\omega_d t) \cos(\omega t) - \sin(\omega_d t) \sin(\omega t) = \cos[(\omega + \omega_d)t]. \tag{4.4}
\]

The SSB modulated signal is sent to the readout circuit followed by down conversion with the original microwave signal \(\cos(\omega t)\) taken from the second output of the vector signal generator. As there is only a single signal generator in the setup, there cannot be any phase drift.

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\[\text{This setup was not used to acquire data presented in this thesis as it was only developed after the data was acquired. However, all setups were modified shortly after due to its convenience.}\]
Chapter 5

Semiconductor-Based Superconducting Qubits

Transmon qubits have over the last decade proven to be promising candidates for scalable quantum computing based on quantum error correction. The experimental implementation is incredibly robust due to the simplicity of the qubit and control circuitry. Furthermore, transmon qubits generally either works or has a ‘catastrophic’ failure resulting in a fast response on device quality. Within a day of measurements the main figures of merit, such as lifetime and coherence time, can be extracted leading to fast optimization loops between fabrication and measurements. In addition, as transmons commonly have one or no tuning parameter, the experimental phase space to be explored for best qubit performance is limited. These factors collected have allowed superconducting qubit performance to increase exponentially over the years leading to state-of-the-art qubits at the border of quantum error correction.

Transmon qubits are commonly based around aluminium tunnel junctions. An alternative approach is based on superconductor-semiconductor Josephson junction taking advantage of the field effect tunability of semiconductors presented in section 3.3. This chapter presents the development of gatemon qubits. The first section introduces the gatemon qubit and has previously been published in [73]. The second part explores the gate fidelities of Clifford gates needed for quantum error correction as well as implementing two-qubit gates. Data presented in this section has previously been published in [118].

5.1 The Gatemon

This section presents data from two gatemon devices, which show similar performance. Except where noted, data are from the first device. The qubit features a single InAs Josephson junction shunted by a capacitance, $C_S$ [17, 67, 119]. The Josephson junction is formed from a molecular beam epitaxy-grown InAs nanowire, $\sim 75$ nm in diameter, with an in situ grown $\sim 30$ nm thick Al shell. The Al shell forms an atomically matched
Al-InAs interface leading to a proximity-induced gap in the InAs core with a low density of states below the superconducting gap (hard gap) [31, 78]. By wet etching away a \( \sim 80 \) nm segment of the Al shell [Figure 5.1A] a weak link in the superconducting shell is formed, creating the Josephson junction [see section 4.1 for details]. A supercurrent leaking through the semiconductor core links the unetched regions and determines the Josephson coupling energy, \( E_J(V_G) \), which can be tuned by changing the electron density in the semiconductor core with a nearby side gate voltage, \( V_G \).

As with conventional transmons, the gatemon operates as an anharmonic LC oscillator with a nonlinear inductance provided by the Josephson junction. The total capacitance of the gatemon qubit \( C_\Sigma \) is determined by the capacitance of the T-shaped Al island to the surrounding Al ground plane, as shown in Figure 5.1C. The gatemon operates with \( E_J \gg E_C \), where the charging energy, \( E_C = e^2/2C_\Sigma \). In this regime, decoherence due to either low-frequency charge noise on the island or quasiparticle tunneling across the Josephson junction is strongly suppressed. For many conducting channels in the wire, the qubit transition frequency is given by \( f_Q = E_{01}/h \approx \sqrt{8E_CE_J(V_G)}/h \). The difference between \( E_{01} \) and the next successive levels, \( E_{12} \), is the anharmonicity, \( \alpha = E_{12} - E_{01} \approx -E_C \). From electrostatic simulations we estimate a charging energy of \( E_C/h \approx 200 \) MHz (\( C_\Sigma \approx 94 \) fF). With this charging energy and \( E_{01}/h = 6 \) GHz we get \( I_c = eE_{01}^2/4E_C h = 45 \) nA (with an effective junction inductance of 7.3 nH), consistent with transport measurements on the same kind of nanowires in Figure 3.7. From microwave spectroscopy of our gatemon, we measure \( \alpha/h \approx 100 \) MHz. The discrepancy between the measured anharmonicity and \( -E_C \) is due to a nonsinusoidal current-phase relation for the nanowire Josephson junction resulting in a reduced nonlinearity in the Josephson inductance [74, 80].

The gatemon is coupled to a \( \lambda/2 \) superconducting transmission line cavity with a bare resonance frequency \( f_C = 5.96 \) GHz and quality factor, \( Q \sim 1500 \). The cavity is used for dispersive readout of the qubit with homodyne detection Figure 5.1D. The frequencies of the microwave signals used to control and readout the qubit are indicated as \( f_Q \) and \( f_C \) respectively. Both the cavity and qubit leads are patterned by wet etching an Al film on an oxidized high-resistivity Si substrate. Nanowires are transferred from the growth substrate to the device chip using a dry deposition technique [114]. During transfer, a PMMA mask ensures nanowires are only deposited on the device inside a \( 85 \) \( \mu \)m \( \times \) 56 \( \mu \)m window where the Josephson junction is fabricated. Following the nanowire shell etch, the nanowire contacts and gate are patterned from Al using a lift-off process with an ion mill step to remove the native Al\(_2\)O\(_3\) prior to deposition. Measurements are performed with the sample inside an Al box mounted at the mixing chamber of a cryogen-free dilution refrigerator with a base temperature < 50 mK [Figure C.1].

To directly measure the qubit-cavity coupling the gatemon is tuned with the gate voltage into resonance with the lowest mode of the cavity. This is the resonant regime of the Jaynes-Cummings Hamiltonian. In Figure 5.2A the cavity response is shown as a function of gate voltage and cavity drive frequency for low driving power. We observe two transmission peaks in the cavity aperiodically modulated by the gate voltage on the gatemon. The aperiodicity is consistent with mesoscopic fluctuations in the conductance.
Figure 5.1:  
A The nanowire Josephson junction integrated into a transmon circuit.  
B The nanowire is contacted at each end and a nearby gate electrode can tune the Josephson energy of the junction.  
C The transmon is formed by a T-shaped island shorted to the surrounding ground plane through the nanowire Josephson junction. The transmon circuit is closed by the capacitance of the island to ground. The island is capacitively coupled to a $\lambda/2$ microwave cavity for readout.  
D Schematic of the gatemon circuit.
of the nanowire junction. The two peaks are the hybridized cavity-gatemon states [66]. Two widely split peaks, Figure 5.2B, indicate a cavity-gatemon in the strong coupling regime with \( g \) larger than both the decay rate of the qubit and the cavity. Off resonance, the qubit-cavity states are weakly hybridized and we only observe one peak, the cavity resonance.

To better estimate the coupling strength \( g \) we extract the peak splitting \( \delta \) for each voltage value with two peaks in 5.2A. The hybridized states, \( f_\pm \), can be calculated from the coupling strength \( g \),

\[
f_\pm = f_Q + f_C \pm \sqrt{(f_Q - f_C)^2 + 4(g/2\pi)^2},
\]

where \( f_C \) and \( f_Q \) are the frequencies of the uncoupled cavity and qubit respectively. By plotting the peak splitting \( \delta = f_+ - f_- = \sqrt{(f_Q - f_C)^2 + 4(g/2\pi)^2} \) as a function of \( f_Q \) as shown in Figure 5.2C we extract a coupling strength \( g/2\pi = 99 \text{ MHz} \). Plotting the data in 5.2A parametrically as a function of the extracted \( f_Q \) the expected avoided crossing of a coupled two-level system is revealed in Figure 5.2D.

To perform coherent operation on the gatemon, we detune it away from the cavity frequency to the dispersive regime. While continuously monitoring the cavity transmission at the cavity frequency we sweep a second microwave tone to drive the qubit. When the qubit drive, the second tone, hits the resonance frequency of the qubit, the qubit is excited into an incoherent superposition of \( |0\rangle \) and \( |1\rangle \) which modulates the monitored cavity transmission. By sweeping the frequency of the qubit drive and the gate voltage we map out the spectrum of the gatemon in Figure 5.3. In the spectrum, we directly observe the aperiodic modulation of the gatemon frequency originating from mesoscopic fluctuations in the nanowire. These fluctuations create local minima and maxima that are first-order insensitive to gate voltage (sweet spots). We also observe discontinuous jumps in the spectrum that we attribute to charge traps near the nanowire changing the charge landscape. Such jumps rarely happen if the gate voltage is restricted to a small voltage range.

Figure 5.4A shows a scan of the qubit spectrum around the sweet spot at 3.4 V. Here spectroscopy is performed by first applying a 2 \( \mu \)s long qubit drive tone and then probing the cavity response to avoid Stark shift in the data. To perform qubit operations on the gatemon we fix the gate voltage at 3.4 V indicated by B. In the top panel of Figure 5.4B the pulse scheme for Rabi oscillations is shown. First, a qubit drive tone of length \( \tau \) rotates the qubit about the X axis and then a readout tone measures the probability for the qubit to be in the \( |1\rangle \) state. The lower part of the main panel shows Rabi oscillations as the qubit is rotated around the Bloch sphere. When the qubit drive is detuned from the resonance frequency the qubit does not fully reach the \( |1\rangle \) state as seen in the amplitude of the oscillations. The rotation axis is the combination of the drive strength along the X axis plus a constant \( \hat{\sigma}_z \) contribution due to the detuning (see equation (3.37)). This also causes a faster oscillation frequency. By sweeping the drive time and drive detuning we can see the effect as the Chevron pattern in the main panel.

While drive pulses around an axis in the XY plane are enough to perform all single-
CHAPTER 5. SEMICONDUCTOR-BASED SUPERCONDUCTING QUBITS

Figure 5.2: A Hybridization of the microwave cavity and gate mon qubit. Extracted qubit frequency and cavity frequency shown as green and blue lines respectively. B Line cut of A indicated by purple arrows. Clearly separated peaks in the transmission. C The vacuum Rabi splitting as a function of extracted qubit frequency. D Parametric plot of the vacuum Rabi splitting as a function of extracted qubit frequency reveals the expected anticrossing of two hybridized states.

Figure 5.3: Spectroscopy measurements of the qubit frequency as a function of gate voltage. Each column is normalized by the value at 3.8 GHz.
Figure 5.4: **A** Spectroscopy of the gatemon. **B** Upper panel shows the pulse sequence for qubit rotations around the X axis on the Bloch sphere. Main panel shows Rabi oscillations as a function of drive time $\tau$ and qubit drive frequency. Lower panel is a line cut at the qubit frequency. **C** The pulse sequence for Z rotations is shown in the upper panel. Main panel shows rotations as a function of drive time $\tau$ and gate pulse amplitude. Lower panel is a line cut at pulse amplitude $\Delta V_G = 20.9$ mV. Normalized state probability, $p_{|1\rangle}$, is calculated from the demodulated cavity response, $V_H$, by fitting Rabi oscillations to an exponentially damped sinusoid of the form $V_H^0 + \Delta V_H \exp(-\tau/\tau_{\text{Rabi}}) \sin(\omega \tau + \phi)$ to give $p_{|1\rangle} = (V_H - V_H^0)/2\Delta V_H + 1/2$. The solid curves in lower panels of A and B are exponentially damped sinusoids.
qubit operations, it is convenient to have fast control of the qubit frequency to perform two-qubit operations. To show that the gate-mom allows fast adiabatic pulses of the qubit frequency we measure the effect on a single qubit. Changing the qubit frequency effectively induces Z rotations on the Bloch sphere. The qubit response is simplified by placing the gate voltage at $3.27 \, \text{V}$ (indicated by C in Figure 5.4A) where the qubit frequency depends linearly on the gate voltage. This ensures that the rotations induced by the change in frequency depend linearly on the gate voltage. To observe rotations about the Z axis, we first rotate the qubit by $\pi/2$ to the equator of the Bloch sphere as shown in the upper panel of Figure 5.4C. Then a gate pulse of length $\tau$ induces a rotation around the Z axis with a frequency proportional to the pulse amplitude (due to the linear energy spectrum). A final $\pi/2$-pulse allows effective readout along the Y axis of the Bloch sphere. In the main panel, we see the rotations that depend on the length of the pulse and the amplitude of the voltage pulse on the gate demonstrating coherent voltage-pulse operations. Furthermore, these operations demonstrate the stability of the gate-mom as the data in the main panel of Figure 5.4B were collected over several hours.

In Figure 5.5 coherence measurements of two gate-mom qubits are shown. The pulse schemes for measuring the lifetime $T_1$ and dephasing time $T_2^*$ are shown in black and blue respectively. Lifetime is measured by varying the delay time between a $\pi$ pulse, which rotates the qubit to the $|1\rangle$ state, and readout. The expected exponential decay with a characteristic time of the $|1\rangle$ state probability is observed. We extract lifetimes $0.56 \, \mu\text{s}$ and $0.83 \, \mu\text{s}$ for sample 1 and 2 respectively.

The dephasing time is measured by placing the qubit on the equator with a slightly detuned $\pi/2$ pulse so that the qubit state precesses around the equator. Readout is performed after a delay time, $\tau$, and a second $\pi/2$ pulse to rotate the Y axis onto the Z axis for readout. We observe the precession of the qubit as the oscillation while the exponential decay with a characteristic time scale $T_2^*$ is due to decoherence. Lifetime enforces an upper limit on the decoherence time of $T_2^* \leq 2T_1$ as the qubit can decay from the superposition state on the equator. For Sample 1 we find $T_2^* = 0.97 \, \mu\text{s}$ very close to the lifetime limit. Sample 2 performs slightly worse with $T_2^* = 0.71 \, \mu\text{s}$. For drifts in the qubit frequency that are constant within the time of the pulse sequence, one can cancel the effect by performing an echo pulse as indicated in red. The pulse effectively reverses the sign on the noise such that noise picked up before the echo pulse is canceled by noise after the pulse. High-frequency noise switching faster than the sequence length will not be canceled by an echo pulse. More elaborate pulse schemes can be used to avoid specific parts of the noise spectrum in the system [120]. We can increase the decoherence time of qubit 2 to $T_{\text{Echo}} = 0.95 \, \mu\text{s}$ with an echo pulse. The fact that we do not reach the lifetime limit indicates that qubit 2 suffers from some high-frequency noise.

Lifetime and coherence times in this first generation of devices is primarily limited by materials loss in the capacitors. Especially a $\sim 87 \, \text{nm}$ thick thermal oxide layer on the silicon substrate leads to losses in capacitors. The oxide layer is initially included to ensure electrical separation between the gate electrodes and the surrounding ground plane. Subsequent studies reveal that a certain distance between contacts and gate electrodes sufficiently insulates the electrodes (for $\sim 1 \, \mu\text{m}$ substrate leakage happens at
Figure 5.5: A Left shows lifetime measurement of sample 1 at point B in Figure 5.4A ($V_G = 3.4$ V). Left side in upper panel shows the pulse scheme for lifetime measurements. A 30 ns $\pi$ pulse rotates the qubit to the $|1\rangle$ state and a wait time $\tau$ before readout is varied. Solid curve is an exponential fit. Right side shows a Ramsey experiment performed by varying a wait time $\tau$ between two slightly detuned 15 ns $\pi/2$ pulses. Solid curve is an exponentially damped sinusoid from which we determine $T^*_2$. B Lifetime and Ramsey experiments are repeated for sample 2 which has $f_Q = 4.426$ GHz ($V_G = -11.3$ V). Furthermore we perform a Hahn echo experiment in red with a $\pi$ pulse inserted between two $\pi/2$ pulses with a varying wait time $\tau$. The phase of the second $\pi/2$ pulse is varied to fit an exponential decay to the extracted amplitude.
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\[ \sim \pm 10 \text{ V on gates} \] allowing for removal of the global thermal oxide. This combined with careful optimization of capacitor loss in resonator Q-factor measurements lead to an almost tenfold increase in lifetime for the second generation \[118\] of gate-mons. Recent studies have demonstrated coherence times of gate-mons approaching that of state-of-the-art transmon qubits \[113\].

5.2 Gate-mon Benchmarking and Two-Qubit Operations

For gate-mon-based quantum computation, we want to quantify the fidelity of qubit gates including two-qubit gates. To demonstrate two-qubit gates the second generation is designed with two capacitively coupled gate-mons \[32\]. Additionally, several improvements are made in the design layout [Figure 5.6] and fabrication. Separate qubit drive lines labelled 'XY control' for individual qubit control, separate \( \lambda/4 \) readout cavities with resonance frequencies \( f_{C1} = 7.81 \text{ GHz} \) and \( f_{C2} = 7.73 \text{ GHz} \) coupled to the same feedline for frequency-multiplexed readout, low loss substrate for reduced capacitor loss, and control circuitry with added crossovers to reduce spurious modes in the ground plane \[110\]. The qubit parameters differ only slightly from the first generation of gate-mons with simulated values of charging energy \( E_C/h = 230 \text{ MHz} \) and qubit-cavity coupling \( g/2\pi \approx 100 \text{ MHz} \). Electrostatic simulations predict a qubit-qubit coupling strength \( J \sim 20 \text{ MHz} \).

With the improvements, we measure almost tenfold improvements for qubit lifetimes up to \( T_1 \sim 5 \mu s \) and decoherence times \( T_2' \sim 4 \mu s \), which can with a single refocusing pulse be increased to \( T_{Ech} \sim 9.5 \mu s \) almost at the limit of \( 2T_1 \). Next, we quantify the fidelity of qubit operations with randomized benchmarking. We expect microwave induced gates to have the same performance as other transmons as the capacitive coupling to the system is independent of the Josephson junction. However, \( Z \) and \( Z/2 \) gates can be induced by baseband pulsing of the junction gate changing \( E_J(V_G) \). We want to verify high fidelity of microwave-induced gates and test the fidelity of qubit gates induced by voltage pulses.

Complete information of a qubit operation can be measured with quantum process tomography \[121\], which describes how any input qubit state is processed by the qubit operation to a qubit output state. However, each process matrix for just a single qubit operation requires measuring 12 independent numbers with high precision\[112\]! Evidently, it is not practical to fully characterize a set of quantum operations due to time constraints. We need a procedure that faithfully estimates the fidelity of gates in an efficient manner.

For this purpose randomized benchmarking \[123\] has been adopted as a standard for quantifying error rates of Clifford gates. Randomized benchmarking is done by measuring the fidelity of a sequence of \( m \) random Clifford gates composing the identity operation. From the decay of the fidelity of random sequences composing the identity operation as a function of \( m \) one can extract an average error rate of all Clifford gates. Furthermore,

\footnote{It is especially difficult to separate the qubit operation fidelity from errors in initialization and measurements.}
interleaved randomized benchmarking, which interleaves a specific Clifford gate between each random gate of a sequence, allows measurement of error probability of individual gates. Random Clifford gates will randomize the qubit state throughout the sequence effectively mapping any noise onto the depolarization channel. The depolarization channel is fully described by a single probability, $1 - p$, that the qubit state is replaced with a fully mixed state \[39\]. The decay rate of fidelity is given by $p^m$ (the probability that the qubit has not been replaced by a mixed state after $m$ operations) and the average gate error is \( r = (1 - p)/2. \) Importantly as the measurement relies on the decay rate of fidelity, it becomes independent of errors in state preparation and measurement (SPAM errors).

Figure 5.7 shows data from interleaved randomized benchmarking on Q2. First, a reference of non-interleaved randomized benchmarking is performed using only microwave induced pulses. Black diamonds are the fidelity of pulse sequences comprising $m$ random Clifford gates, $C$, followed by a recovery pulse, $C_R$, such that $C_R(C)^m = I$. As the full sequence composes the identity operation, the fidelity can be measured as the $|0\rangle$-state population. Each Clifford gate is generated by one or more Gaussian shaped microwave pulses with standard deviation $\sigma = 7$ ns and truncated to a full gate time of $t_g = 28$ ns. These pulses were optimized using AllXY pulse sequences \[106\] and randomized benchmarking sequences \[108\]. The fidelity decays is fitted to $A p^m_{\text{ref}} + B$ with $A = 0.53$ and $B = 0.42$ accounting for SPAM errors. From $p_{\text{ref}} = 0.981$ we extract an average single-qubit error rate of $r_{\text{ref}} = (1 - p_{\text{ref}})/(2 \times 1.875) = 0.5 \pm 0.07 \%$, where the factor 1.875 is the average number of single qubit gates per Clifford gate.

For interleaved randomized benchmarking of gate $G$ we measure the fidelity of pulse

\[ \text{Figure 5.6: Optical image of a two-qubit gatemon device. The two qubits are coupled to individual } \lambda/4 \text{ cavities. Coherent operations are performed by drive lines coupled capacitively to the gatemons.} \]
sequences comprising of $C_R(GC)^m = I$, where $C$ again is random Clifford gates and $C_R$ is the recovery pulse such that the total sequence composes the identity operation. Measuring the decay of fidelity as a function of $m$ we again fit the decay to $Ap_G^m + B$ and extract $p_G$ for each gate tested. From $p_g$ and $p_{ref}$ the average gate error for gate $G$ is given by $r_g = (1 - p_G/p_{ref})/2$. Inset in Figure 5.7 displays the error rates of individual gates. Qubit gates induced by voltage pulses, $Z$ and $Z/2$, are performed by a 28 ns square pulse and reaches error rates of $r_Z = 0.35 \pm 0.19\%$ and $r_{Z/2} = 0.18 \pm 0.15\%$ consistent with the lifetime limit on gate error: $r_{limit} = t_g/3T_1 = 0.3\%$ \([124]\), where $T_1 = 3.1 \mu s$ at the measurement point. This demonstrates that high fidelity gates can be performed with gate voltage pulses in gateon qubits.

Next, we investigate coherent two-qubit operations. The qubit-qubit coupling is observed in two-spectroscopy as an avoided crossing as the Q1 is swept through resonance with Q2 [Figure 5.8A]. To demonstrate that we have coherent control of the two-qubit system we perform $i$SWAP operations. The applied pulse sequence is shown in 5.8B. With the qubits initially off-resonance a single $\pi$ pulse excites Q2 to the $|1\rangle$ state while Q1 is left in the ground state. A gate voltage pulse with amplitude $\Delta V_2$ on Q2 brings the qubits diabatically into resonance for a time $\tau$ before bringing the system back for read-out. The excitation initially on Q2 begins to oscillate between the hybridized qubits with a frequency $J/\pi$. The $|1\rangle$ state probability in Q1 after an $i$SWAP operation is mapped out as a function of waiting time $\tau$ and pulse amplitude $\Delta V_2$ in Figure 5.8C. A chevron pattern is observed as the excitation coherently swaps between Q1 and Q2. A similar plot is obtained for measurements of the $|1\rangle$ state probability of Q2 which is inverted compared to 5.8C. In 5.8D a line trace of both measurements is shown demonstrating the excitation swapping between the two qubits. From the oscillations, we extract a
coupling strength of 17.8 MHz.

The preferred two-qubit gate for quantum algorithms is a controlled phase gate $c^{\pi}_Z$ gate presented in section 3.5. To demonstrate the effect of the gate Q1 is used as control qubit in either state $|0\rangle$ or $|1\rangle$ shown in blue and red respectively in Figure 5.9A. First Q2 is placed on the equator of the Bloch sphere by a $X/2$ pulse to detect rotations around the Z axis. Then a voltage pulse on Q2 brings the system close to the $|20\rangle - |11\rangle$ anticrossing acquiring a phase shift conditional on the state of Q1. Lastly, the acquired phase is measured by varying the angle of a $\pi/2$ pulse before readout. The pulse sequence is adjusted such that the dynamical phase acquired by Q2 due to the change in frequency is a multiple of $2\pi$. Performing the sequence with the control qubit in $|0\rangle$ and $|1\rangle$ we find the conditional phase difference. Figure 5.9B shows the $\pi$ phase dependence of the control qubit as desired for the $c^{\pi}_Z$ gate.

To estimate the gate fidelity we perform interleaved single-qubit randomized benchmarking treating the $c^{\pi}_Z$ gate as a single qubit $Z$ gate as shown in 5.9C. The resulting gate fidelity of $c^{\pi}_Z$ with the state of the control qubit randomized between $|0\rangle$ and $|1\rangle$ is $r = 9 \pm 2 %$ [Figure 5.9D]. Fixing the state of the control qubit leads to similar gate fidelities as shown in the figure inset. We estimate a 4 % error rate due to qubit relaxation and attribute the remaining 5 percentage points to leakage into state $|20\rangle$. 

Figure 5.8: 

A An avoided crossing is observed in spectroscopy measurements as the frequency of Q1 is being swept through the frequency of Q2. 

B Pulse sequence for mapping the qubit-qubit coupling in time domain. Q2 is excited by a $\pi$ pulse followed by a gate pulse with amplitude $\Delta V_2$ and width $\tau$. 

C Swap oscillations as a function of $\Delta V_2$ and width $\tau$. 

D Line cut of C with the gate pulse bringing the qubits into resonance for time $\tau$. 

The pulse sequence is adjusted such that the dynamical phase acquired by Q2 due to the change in frequency is a multiple of $2\pi$. The state of the control qubit is randomized between $|0\rangle$ and $|1\rangle$, and the conditional phase difference is measured.
Figure 5.9: A Pulse sequence to probe phase shift of controlled phase gate $c_{Z}^{\pi}$. Control qubit is placed in either state $|0\rangle$ or $|1\rangle$ shown in blue and red respectively. To observe a phase shift on target qubit Q2 it is placed on the equator by a $X/2$ pulse before the $c_{Z}^{\pi}$ gate is performed. Lastly a $\pi/2$ gate along an axis whose phase $\theta$ is varied. B Probability of Q2 in state $|1\rangle$ as a function of $\theta$. C Pulse scheme for interleaved randomized benchmarking of the controlled phase gate $c_{Z}^{\pi}$. D Fidelity of interleaved, single-qubit randomized benchmarking. Inset shows extract gate errors dependent on the state of the control qubit.

5.3 Conclusion

In conclusion, we have demonstrated a novel semiconductor-based superconducting qubit based on a field effect tunable Josephson junction. Universal single-qubit control is achieved with all-microwave control and with randomized benchmarking we measure 99.5% average fidelity for Clifford gates limited by qubit coherence times. Crucially, nanosecond voltage pulses on the field effect tunable Josephson junction induces qubit frequency modulation without degradation in qubit coherence. This allows implementation of controlled-phase, two-qubit operations forming a sufficient gate set for quantum error correction.

Gateemon qubits offer an alternative all-electrical approach to tunable superconducting qubits alleviating the need for milliampere currents required for conventional flux controlled superconducting qubits. Recent work has demonstrated the feasibility of top-down wafer-scale fabrication of gateemon qubits based on proximitized two-dimensional electron gas paving the way for readily scalable gateemon circuits [76]. Furthermore, state-of-the-art gateemon qubits have demonstrated coherence times approaching that of conventional Al/AlO$_{x}$/Al tunnel junctions [113] putting gateemon qubits in the range of quantum error correcting. Additionally, field effect tunable Josephson junction presents a new circuit element that might enable novel circuits such as tunable couplers [38, 125].
Chapter 6

A Superconducting 0-$\pi$ Qubit Based on High Transmission Josephson Junctions

Topological protection can be engineered at the device level by designing a Hamiltonian performing passive quantum error correction. Such a Hamiltonian is described by multi-qubit terms given by the stabilizers of the error correcting code being implemented. In this Chapter we experimentally investigate a 0-$\pi$ qubit, which is a fundamental building block for topologically protected qubit [29, 126] with protected quantum operations [127]. It is realized in a superconducting circuit element with a $\pi$-periodic, double-well potential in the superconducting phase difference, $\varphi$. The double-well potential forms two protected, degenerate ground states separated by the parity of Cooper pairs.

Recent experimental studies have realized circuit elements generating $\cos(2\varphi)$ potentials [35, 36] in rhombi structures through interference effects between four equally sized aluminium tunnel junctions [128]. However, fabrication variations in the size of AlO$_x$-based Josephson junction elements lift the degeneracy of the lowest two states, limiting the qubit protection. We present a simplified design for this fundamental $\cos(2\varphi)$ building block using hybrid, high-transmission superconductor-semiconductor Josephson junctions. Our approach takes advantage of their non-cosine energy-phase relation and in situ voltage tunability to precisely define a 0-$\pi$ qubit circuit.

The circuit for the semiconductor-based 0-$\pi$ qubit is shown in Figure 6.1(a). The transmonlike geometry consists of a superconducting island with charging energy, $E_C$, connected to ground through two nanowire Josephson junctions arranged in a superconducting quantum interference device (SQUID) configuration. Each junction is controlled using the gate voltage $V_k$ ($k \in \{1, 2\}$). We model the Josephson junctions using short junction theory where the Josephson effect is characterized by a set of transmission coefficients of transport channels in the normal section, $T^{(k)} = \{\tau_i^{(k)}\}$ [129].

The energy-phase relation of the junction is then given by summing over the energies
CHAPTER 6. A SUPERCONDUCTING 0-π QUBIT BASED ON HIGH TRANSMISSION JOSEPHSON JUNCTIONS

Figure 6.1: The 0-π qubit. (a) Circuit schematic of the 0-π qubit formed by high transparency, semiconductor Josephson junctions in a SQUID shunted by a large capacitor. (b-c) Energy-phase relation of the SQUID for different transmission coefficients (b) and different loop asymmetries (c). (d) False color optical image of the large island (blue) forming one side of the shunting capacitor. (e-f) False color scanning electron micrographs. (f) A small segment of the Al shell on an InAs nanowire is etched away to form a semiconductor Josephson junction. A nearby electrostatic gate (red) allows tuning of the electron density in the junction.
of each channel,

\[ U_k(\varphi_k) = -\Delta \sum_{\tau \in T^{(k)}} \sqrt{1 - \tau \sin^2(\varphi_k/2)} , \]

where \( \Delta \) is the superconducting gap and \( \varphi_k \) is the superconducting phase difference across the Josephson junction. The full system is modeled by the Hamiltonian

\[ H = 4E_C \hat{n}^2 - U_1(\dot{\varphi}) - U_2(\dot{\varphi} - 2\pi \Phi/\Phi_0) , \]

where \( \Phi \) is the applied flux through the SQUID loop and \( \Phi_0 = h/2e \) is the superconducting flux quantum. For identical junctions at one-half flux quantum, \( \Phi = \Phi_0/2 \), odd harmonics in the Hamiltonian potential, \( -U_1(\dot{\varphi}) - U_2(\dot{\varphi} - 2\pi \Phi/\Phi_0) \), are suppressed, leaving even harmonics of the potential. Figure 6.1(b) shows the even harmonics of the potential, present in high transmission junction with a dominant \( \cos(2\hat{\varphi}) \) term, forming the characteristic \( \pi \)-periodic potential of a 0-\( \pi \) qubit with degenerate ground states. In charge basis the degeneracy originates from the Josephson current across the SQUID occurring only in units of 4e charge, that is, pairs of Cooper pairs. The suppression of single Cooper pair transport results in the qubit having doubly degenerate ground states that differ by the parity of Cooper pairs on the island. The height of the potential barriers separating the two wells scales with the symmetry, \( \alpha \), of the SQUID. Breaking the symmetry, shown in Figure 6.1(c), will lower one barrier and raise the other, recovering the single-well potential of a single-junction gate in the limit \( \alpha \to 0 \). In the intermediate regime, the potential resembles that of a flux qubit. Related work with nanowire SQUID transmons has measured double-well potentials in similar circuits [74].

The device is shown in Figure 6.1(d-f). A large T-shaped island (blue), embedded in a 100 nm thin aluminium ground plane, forms the shunting capacitor of the transmon-like circuit. From simulations we estimate the charging energy of the capacitor to be \( E_C \sim 235 \text{ MHz} \). The sample is fabricated on a high-resistive silicon substrate. Two hexagonal InAs nanowires grown by molecular beam epitaxy with a \( \sim 10 \text{ nm} \) thick epitaxial aluminium shell on two facets are placed in between the island and the ground plane. Prior to contacting the nanowire a light argon mill was applied to remove the native oxide of InAs. Josephson junctions are formed by etching away a small segment of the aluminium shell of the nanowire. Nearby electrostatic gates (red) tune the electron density and hence the transmission of conduction channels in the Josephson junctions. By applying a current to a transmission line shorted to the ground plane near the SQUID a small magnetic field is generated allowing control of the flux, \( \Phi \), penetrating the loop. The island is capacitively coupled to a \( \lambda/4 \) cavity with coupling strength \( g/2\pi \sim 80 \text{ MHz} \) in the transmon regime. The system is driven by microwave excitations applied to a nearby open transmission line. The sample is loaded in an Al box placed inside a magnetic shield and measured in a dilution refrigerator at \(< 50 \text{ mK} \) [Details in C.3].

In Figure 6.2(a) we first probe the resonance frequency of the \( \lambda/4 \) cavity as a function of flux, \( \Phi \), through the SQUID. A vacuum-Rabi splitting is visible as the cavity state hybridizes with the qubit state (red line) when on resonance. Several additional qubit states are weakly coupled to the cavity giving rise to smaller anticrossings. To perform
Figure 6.2: Qubit spectroscopy as a function of flux, $\Phi$, at voltages $V_1 = 1.4$ V and $V_2 = -0.445$ V. (a) Resonance frequency of the readout cavity as the qubit energy is modified by flux. Solid and dotted lines are from fit to data in (c). (b) Two-tone spectroscopy of the qubit transition frequencies. An average of each column has been subtracted. (c) Extracted transition frequencies from (b) with solid lines the results of a fit to Eqn. (6.2). Cartoons above shows the fitted potential at different values of $\Phi$. Gray dotted lines are multi-photon transitions due to simultaneous readout and drive tones.
two-tone spectroscopy a readout tone, adjusted at each point in flux to the cavity frequency extracted from Figure 6.2(a), is monitored while a second drive tone is swept in frequency to excite energy states. Figure 6.2(b) shows the monitored signal with peaks at the transition frequencies of the qubit system excited by the drive tone.

At $\Phi = 0.63 \Phi_0$, away from one-half flux quantum, we observe two transition frequencies near 8.4 GHz closely resembling the spectrum of a transmon qubit with the higher frequency transition being $f_{01}$ (red) and the lower a 2-photon excitation $f_{02}/2$ (blue). As the flux is tuned closer to one-half flux quantum the frequency of $f_{01}$ sharply drops with several strongly dispersing transitions. The horizontal, non-dispersing lines in the spectrum, which we interpret as on-chip resonances, amplify the readout response as transition frequencies cross.

To model the spectrum we extract excitation frequencies of $f_{01}$, $f_{02}$, $f_{02}/2$, and $f_{12}$ shown as colored circles in Figure 6.2(c). The extracted frequencies are fitted to the model Hamiltonian in Eqn. (6.2) by numerically simulating its eigenenergies with $\Delta/h = 45$ GHz [78] (details given in Section 6.1). The results are plotted as solid lines in Figure 6.2(c). From the fit we extract charging energy $E_C = 280$ MHz and sets of transmission coefficients for each junction $T^{(1)} = \{1, 1, 0.553, 0\}$ and $T^{(2)} = \{0.945, 0.14, 0.14\}$. Cartoons above Figure 6.2(c) plot the Josephson potential of the SQUID at different values of $\Phi$. At $\Phi = 0.5 \Phi_0$ the potential is a symmetric double-well potential due to the high transmission of the nanowire Josephson junction. Moving away from $\Phi = 0.5 \Phi_0$ the potential is tilted causing $f_{01}$ to sharply rise in energy. Further tilting the potential results in a single well and the transmon spectrum of a weakly anharmonic oscillator is recovered.

We can match other transitions (gray lines) to multi-photon excitations due to simultaneous readout and drive tones. These transition frequencies are calculated by subtracting an integer number of the cavity resonance frequency, $f_r$, from the fitted spectrum. Small differences between model and data might be explained by small modifications to the measured resonance frequency due to the AC Stark shift of transition frequencies.

Next, we study the effect of tuning the gate voltages for each nanowire Josephson junction. As highlighted in Figure 6.1(c), the relative size of the two Josephson junctions can strongly modify the qubit potential. First, we tune the gate voltages so that the two junctions are of similar coupling strength to form a double-well potential [Figure 6.3(a)]. A single transition frequency is present in the spectrum with two smaller anticrossings at $\Phi \sim 0.48 \Phi_0$ and 0.52 $\Phi_0$. At these points in flux, the potential is tilted such that states $|1\rangle$ and $|2\rangle$, living in separate, wells are on resonance. A small coupling across the middle barrier couples the two states causing an avoided crossing between $f_{01}$ and $f_{02}$. At flux points closer to $\Phi = \Phi_0/2$ the visible transition near 8 GHz is an excitation from the ground state to the next lowest energy state, $|2\rangle$, of the same well. Microwave-induced transitions between wells are suppressed due to a missing overlap of wavefunctions between states in separate wells. The spectrum is reminiscent to that of a heavy fluxonium [130, 131].

Using the measured transition frequencies, $E_C = 295$ MHz, and $\Delta = 45$ GHz we extract the transmission coefficients $T^{(1)} = \{1, 1, 0.605, 0\}$ and $T^{(2)} = \{0.991, 0.758, 0.574, 0.011\}$. 

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Figure 6.3: Voltage control of middle barrier. (a) Gate voltages tuned towards a balanced regime with the two junction of similar coupling strength, $V_1 = 1.2$ V and $V_2 = -0.12$ V. (b) Qubit potential and wave functions two lowest energy states extracted from fit to data in (a) at $\Phi = \Phi_0/2$. (c) The charge distribution of the two lowest energy states. (d) Gate voltages tuned towards an unbalanced regime with one junction much smaller than the other, $V_1 = 1.241$ V and $V_2 = -0.386$ V. (e,f) Potential and wavefunctions of two lowest energy states extracted from fits to (d).
CHAPTER 6. A SUPERCONDUCTING 0-π QUBIT BASED ON HIGH TRANSMISSION JOSEPHSON JUNCTIONS

Figure 6.4: Coherent control. Rabi oscillations in (a) the transmon regime with $\Phi = 0$ and $f_{\text{Drive}} = 7.911$ GHz and (b) a tilted 0-π qubit regime with $\Phi = 0.512 \Phi_0$ and $f_{\text{Drive}} = 5.725$ GHz. Cartoons show the qubit potentials with lowest energy states with voltages fixed at $V_1 = -1.25$ V and $V_2 = -0.445$ V. Solid line in (a) is a fit to an exponentially decaying sinusoidal function. In (b) the fit function has an additional exponentially decaying offset (dashed line).

At $\Phi = \Phi_0/2$ the potential forms a symmetric double well with minima at $\pm \pi/2$. Two nearly degenerate ground states of the system are formed by the bonding and antibonding eigenstates of the potential [Figure 6.3(b)]. In Figure 6.3(c) the wavefunctions of the two ground states are plotted in charge basis clearly visualizing the two states as even or odd numbers of Cooper pairs.

We now tune the gate voltage such that one junction much smaller than the other. Figure 6.3(d) shows the qubit spectrum as a function of $\Phi$. A similar fitting procedure yields $T^{(1)} = \{1, 1, 0.308, 0.308\}$ and $T^{(2)} = \{0.891, 0.112, 0.112\}$. Modeling the potential and its two lowest energy states [Figure 6.3(e,f)] we find a harmonic oscillator with a small perturbation giving a positive anharmonicity similar to a flux qubit [15]. We hence demonstrate by means of voltage and flux in situ tunability between widely different qubit regimes: A transmon, 0-π qubit, and a flux qubit.

The simulations find good agreement with data by varying only the transmission coefficients in each junction giving confidence that the model closely resembles the system. However, we find a discrepancy between the charging energy simulated from electrostatics and from fitting the data given by 235 MHz and 280 MHz respectively. This could be due to the assumption of fixed gap energy, $\Delta$, for all channels in both junctions or other simplifications in the model such as not accounting for charge renormalization at transmissions near unity [86].

In Figure 6.4 we perform time-domain measurements of the qubit in two different regimes at fixed gate voltages. First, we set $\Phi = 0$ such that the qubit is in a transmon regime with a single-well potential. Applying a drive tone at the qubit resonance fre-
quency for a time $\tau$ followed by readout, we observe Rabi oscillation as expected for a transmon qubit. Next, we tune the flux to $\Phi = 0.512 \Phi_0$ where the qubit potential forms a tilted double well shown in Figure 6.4(b). A very weak matrix element between $|0\rangle$ and $|1\rangle$ forbids direct Rabi oscillations between these states. Instead, we apply a drive tone at the $|0\rangle$-$|2\rangle$ transition frequency inducing Rabi oscillations between $|0\rangle$ and $|2\rangle$ state [Figure 6.4(b)]. The oscillations appear around an exponentially decaying offset (black dashed line) which we interpret as decay from the $|2\rangle$ state to the $|1\rangle$ state, leading the population to be trapped in the $|1\rangle$ state for large $\tau$.

To probe the protection offered by the double-well potential we measure the lifetime of the qubit in each regime [Figure 6.5]. In the transmon regime (blue), we measure the lifetime by varying a wait time $\tau$ between a $\pi$-pulse and readout. The data is fitted to an exponential decay from which we extract lifetime $T_1 = 0.6 \mu s$. In the double well regime we cannot perform a direct $\pi$-pulse from state $|0\rangle$ to state $|1\rangle$ as the $|0\rangle - |1\rangle$ transition is forbidden. Instead we drive $|0\rangle - |2\rangle$ for $3 \mu s$ to initialize the state in $|1\rangle$ followed by a measurement delayed by a wait time $\tau$. Due to a small part of the qubit state left in $|2\rangle$, we observe two superimposed exponential decays from which we extract lifetimes $T_1^{(1)} = 7.2 \mu s$ and $T_1^{(2)} = 1.2 \mu s$.

Enhanced lifetimes in the double-well regime are predicted due to a suppressed charge matrix element $\langle 0 | \hat{n} | 1 \rangle$ [see section 6.1]. From the model, we calculate a ratio of 18 between the charge matrix elements in the two regimes. We speculate that lifetime improvement is limited due to other decay channels such as quasiparticles or residual
subgap resistance in the nanowire Josephson junctions. In tunnel probe spectroscopy it has been shown that subgap states can be tuned by the electron density of the nanowire [132, 133]. A low electron density might be reached with larger gates tuning the full length of the nanowire as well as shorter etched Josephson junctions.

Further work on 0-π qubits based on high-transmission nanowire Josephson junctions is needed to develop a robust readout scheme. The present readout scheme based on capacitively coupled resonators cannot distinguish the two ground states in the symmetric double-well regime as the two Cooper-pair parity states gives an identical push on the readout resonator. This can possibly be overcome by parametrically driven readout [134] or by dynamically detuning from the protected regime for readout. Future experiments on 0-π qubits could include coupling to an LC resonator with a superinductance to perform protected qubit rotation [127].

In summary, we have studied a novel superconducting-circuit architecture based on highly transmissive semiconductor-superconductor Josephson junctions with in situ tunability between widely different qubit regimes. In a double-well potential, near the symmetric 0-π regime, we measure enhanced lifetimes indicating a protected qubit. From fits to the qubit spectra we conclude that the semiconductor-nanowire Josephson junctions are dominated by a few conduction channels with transmission coefficients close to unity consistent with recent studies [80].
6.1 Supplementary Information

Numerical simulation of eigenstates and fitting of energy spectra

This section discusses the numerical simulation of eigenstates and how the fit is performed.

We want to fit the data to the Hamiltonian of two high-transmission Josephson junctions in a SQUID geometry given by:

$$H = 4E_C n^2 - \sum_{\tau \in T(1)} \Delta \sqrt{1 - \tau \sin^2(\varphi/2)} - \sum_{\tau \in T(2)} \Delta \sqrt{1 - \tau \sin^2[(\varphi - \phi)/2]},$$  \hspace{1cm} (6.3)

where $\phi = 2\pi \Phi/\Phi_0$. For numerical simulations of the Hamiltonian, we first rewrite the Hamiltonian in charge basis. The Josephson junctions given in phase basis is transformed into charge basis by performing a discrete Fourier transform of the energy-phase relation:

$$- \sum_{\tau \in T} \Delta \sqrt{1 - \tau \sin^2[(\varphi - \phi)/2]} = - \sum_{k=1}^{\infty} E_k \cos[k(\varphi - \phi)]$$  \hspace{1cm} (6.4)

$$= - \sum_{k=1}^{\infty} E_k e^{ik(\varphi - \phi)} + e^{-ik(\varphi - \phi)}$$  \hspace{1cm} (6.5)

$$= - \sum_{k=1}^{\infty} E_k e^{-ik\phi} |n + k\rangle + e^{ik\phi} |n + k\rangle \langle n|,$$  \hspace{1cm} (6.6)

The Hamiltonian can then be written in the charge basis as:

$$H = 4E_C n^2 |n\rangle \langle n| - \sum_{k=1}^{\infty} E_k^{(1)} |n\rangle \langle n + k| + |n + k\rangle \langle n|$$

$$- \sum_{k=1}^{\infty} e^{-ik\phi} E_k^{(2)} |n\rangle \langle n + k| + e^{ik\phi} |n + k\rangle \langle n|$$

$$= 4E_C n^2 |n\rangle \langle n| - \sum_{k=1}^{\infty} E_k^{(1)} + e^{-ik\phi} E_k^{(2)} |n\rangle \langle n + k| + E_k^{(1)} + e^{ik\phi} E_k^{(2)} |n + k\rangle \langle n|,$$ \hspace{1cm} (6.7)

where $E_k^{(1)}(T(1), \Delta)$ and $E_k^{(2)}(T(2), \Delta)$ are the Fourier components of each Josephson junction. In a basis of charge states, $|n\rangle$, given by $\mathcal{V} = \{|\ldots, 2\rangle, |1\rangle, |0\rangle, |-1\rangle, \ldots\}$, the Hamiltonian is the matrix:

$$H = \begin{bmatrix}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & 16E_C & -E_1^{(1)} - e^{i\phi} E_1^{(2)} & -E_1^{(1)} - e^{i2\phi} E_1^{(2)} & -E_1^{(1)} - e^{i3\phi} E_1^{(2)} & \cdots \\
\cdots & -E_2^{(1)} - e^{-i\phi} E_2^{(2)} & 4E_C & -E_2^{(1)} - e^{i\phi} E_2^{(2)} & -E_2^{(1)} - e^{i2\phi} E_2^{(2)} & \cdots \\
\cdots & -E_3^{(1)} - e^{-i2\phi} E_3^{(2)} & -E_1^{(1)} - e^{-i\phi} E_1^{(2)} & 0 & -E_1^{(1)} - e^{i\phi} E_1^{(2)} & \cdots \\
\cdots & -E_3^{(1)} - e^{-i3\phi} E_3^{(2)} & -E_2^{(1)} - e^{-i2\phi} E_2^{(2)} & -E_2^{(1)} - e^{-i3\phi} E_2^{(2)} & 4E_C & \cdots \\
\cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}$$ \hspace{1cm} (6.8)

For numerical simulations, we truncate the Hilbert space 41 states.
(\mathcal{V} = \{|20\rangle, \ldots, |1\rangle, |0\rangle, |−1\rangle, \ldots, |−20\rangle\}) and set \(E_k = 0\) if \(E_k < 1\) MHz. For a given value of \(\phi\), eigenenergies \(E_i\) and eigenvectors \(\psi_i(n)\), where \(|i\rangle\) refers to the \(i\)'th eigenstate of the matrix, are found numerically with \texttt{numpy.linalg.eig()} in Python. Transition frequencies of the model at \(\phi\) are readily calculated as the energy differences of the sorted set of eigenenergies \((f_{01} = (E_1 - E_0)/h)\). Eigenvectors of the matrix are wavefunctions of quantum states in charge basis as plotted in Figure 6.3 of the main text. The wavefunctions in phase basis are calculated from the relation \(\psi_i(\phi) = \sum_n e^{in\phi}\psi_i(n)\).

The charge matrix elements are computed as \(|k\rangle\langle n| = \sum_{n=−20}^{20} \psi_k(n)\psi_i^\dagger(n)\).

To fit the data we use \texttt{scipy.optimize.least_squares()} to find the sets of transmissions \(T^{(1)}\) and \(T^{(2)}\) (\(\Delta\) is fixed) that minimizes the differences between numerically simulated transition frequencies and measured transition frequencies for all measured values of \(\phi\).

**Energy spectrum and matrix elements for Figures 6.4 and 6.5**

Figure 6.6 shows spectroscopy data used to extract potentials plotted in Figures 6.4 and 6.5. Figure 6.7 shows calculated charge matrix elements for the fitted model.

Figure 6.6: Energy spectrum for gate voltages at \(V_1 = −1.25\) V and \(V_2 = −0.445\) V.
Figure 6.7: Charge matrix elements for gate voltages at $V_1 = -1.25$ V and $V_2 = -0.445$ V.
Chapter 7

High field compatible transmon circuit

Topological materials present an exciting direction to a scalable, topological quantum computer [135, 136]. Recent studies have provided compelling evidence of Majorana fermions in proximitized semiconducting nanowires with strong spin-orbit coupling [21, 22, 137, 138]. A controlled coupling of Majorana fermions, projecting the protected qubit state into a measurable fermion parity, is a cornerstone in several Majorana-based qubits [136, 139]. Direct coupling of Majorana fermions on separate superconductors gives rise to the fractional Josephson effect. The fractional Josephson coupling might be detected by embedding it into a well-known transmon circuit [140, 141], which provides well-established measurement techniques. In this Chapter, we present a high-field compatible transmon circuit, necessary to enter the topological phase, based on a single superconductor-semiconductor nanowire capable of hosting Majorana fermions. We show that the coherence of the transmon circuit is insensitive to low magnetic fields and survives up to $B = 1$ T sufficient for Majorana fermions.

Transmon qubits consist of a Josephson junction shunted by a large capacitor. The system exhibits harmonic voltage oscillations carried by Cooper pairs. Single electrons, quasiparticles, rarely tunnel across the Josephson junction leading to a metastable, parity degree of freedom changed by single-electron tunneling events. A fractional Josephson effect from two coupled Majorana fermions will introduce a dissipationless, coherent transfer of single electrons. The single-electron supercurrent couple the even and odd parity sectors of the transmon leading to a significant change in the measured spectrum. The Hamiltonian of a transmon qubit with a Majorana coupling, a Majorana transmon, is given by [140]:

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos(\hat{\phi}) + 2E_M i\hat{\gamma}_2\hat{\gamma}_3 \cos(\hat{\phi}/2), \quad (7.1)$$

where $E_M$ is the coupling strength between Majorana fermions $\hat{\gamma}_2$ and $\hat{\gamma}_3$ with a superconducting phase difference $\varphi$. The product $P = i\gamma_2\gamma_3 = \pm 1$ is the fermion parity. Figure 7.1A shows the $4\pi$-periodic potential of the Majorana transmon with $E_J > E_M$. 

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Figure 7.1: A Potential of the Majorana transmon with \( E_J > E_M \). Lowest energy states (not to scale) are shown with the possible single-photon transitions. B Charge dispersion of transitions shown in A for \( E_C/h = 400 \text{ MHz} \), \( E_J/E_C = 27 \), \( E_M/h = 0.5 \text{ MHz} \), and linewidth \( k = 50 \text{ kHz} \). In the transmon limit \( E_J/E_C \gg 1 \) the dispersion flattens out and \( E_M \) introduces a splitting between intra-well transitions B. B is adapted from [140].

and its lowest energy states (not to scale for clarity). Inter-well transitions denoted A and C separates by \( \pm 2E_M \) from intra-well transitions denoted B due to the coupling of parity sectors in the transmon circuit. For \( E_J \gg E_C \) the dispersion flattens out and \( E_M \) is visible as a splitting of intra-well transitions B due to a modification to the harmonic approximation of each well from the fractional Josephson coupling: \( E_J \cos(\varphi) \pm E_M \cos(\varphi/2) \). In this regime, inter-well transitions A and C are suppressed due to non-overlapping wavefunctions.

A topological nanowire with a small break in the aluminium shell as shown in Figure 7.2A is expected to host four Majorana fermions: one at each end and two on each side of the junction. Large plunger electrodes, \( V_{\text{pl}} \) and \( V_{\text{rl}} \), are used to tune the chemical potential of the nanowire into a topological regime. With a third electrode, \( V_{\text{cut}} \), at the junction one can open or close the coupling between Majorana fermions \( \gamma_2 \) and \( \gamma_3 \). In addition, as observed for gatemons, the junction will form a highly coherent Josephson coupling allowing a single multichannel nanowire to mediate both the trivial Josephson coupling as well as the topological Majorana coupling present in Equation (7.1). To turn on the fractional Josephson effect, we need to bring the system into a topological phase by tuning the chemical potential and magnetic field.

Figure 7.2B shows an InAs nanowire with a diameter of \( \sim 100 \text{ nm} \) with one side covered by a 7 nm thick aluminium shell. It is placed on NbTiN bottom gates using a micromanipulator. A small part of the aluminium shell is etched away using a wet etch to form a Josephson junction. The nanowire is connected to a T-shaped qubit island, with simulated charging energy \( E_C = 230 \text{ MHz} \), and the surrounding ground plane [Figure 7.2C]. A light RF mill is used to remove the native oxide of InAs before sputtering of NbTiN contacts. The qubit island is capacitively coupled to a \( \lambda/2 \) cavity with resonance frequency \( \sim 4.95 \text{ GHz} \) for readout and microwave control. The cavity, qubit island, and bottom gates are fabricated for low loss and high-field compatibility in 20 nm NbTiN [77, 142] using e-beam lithography and chlorine-based dry-etch process.
A high density of flux trapping holes is used to trap any flux vortices penetrating the thin NbTiN film. NbTiN crossovers tie ground planes to avoid parasitic modes on the chip. Local 5 nm HfO$_2$ deposited with ALD before the NbTiN deposition ensures no leakage between closely spaced gates through the silicon substrate. A second HfO$_2$ layer 15 nm thick on top of bottom gates acts as gate dielectric between bottom gates and nanowire. On-chip LC-filters (not shown) on each gate electrode suppress microwave dissipation through the capacitively coupled gates. A second qubit with no plunger gates is coupled to the same resonator (not shown) but all data presented is from the qubit shown. The sample is placed inside a CuBe box filled with microwave absorbing Eccosorb foam to reduce microwave and infrared radiation. The box is mounted in a dilution refrigerator with base temperature $< 50$ mK (see Figure C.4 for a schematic of setup).

7.1 Coherent Control up to 1 T

First, we investigate the qubit behavior in a magnetic field parallel to the nanowire with two-tone spectroscopy. During two-tone spectroscopy, the cavity resonance is first measured for each magnetic field value to account for changes in the cavity resonance. It is crucial to not use aluminium on-chip bond wires to connect ground planes as these cause a large amount of dissipation above the critical field of aluminium. Electrodes spaced $\sim 1$ µm apart on bare, high-resistive silicon will leak at $\sim \pm 10$ V at base temperature.
Figure 7.3: Two-tone spectroscopy as a function of magnetic field. Changes in background signal are due to adjustments to qubit drive power to account for varying lifetime and detuning from the readout cavity. Data around 0.58 T omitted due to a mistake in setup during data acquisition. An average is subtracted from data at each magnetic field value.
Appendix Figure B.1]. Any out-of-plane magnetic field on order of 10 $\mu$T will modify the resonance frequency of the cavity, but we observe no degradation of resonator $Q$ factor or qubit lifetimes. Figure 7.3 shows the qubit frequency as a function of magnetic field up to 1 T. The qubit frequency exhibits a lobe structure with minima at $B \sim 0.225$ T and $B \sim 0.675$ T. We interpret this as a suppression of the induced gap in the semiconductor due to interference effects in the cross-section of the nanowire. The current density in the semiconductor is mostly confined to the surface of the nanowire forming a superconducting cylinder in a magnetic flux. The magnetic field will induce a supercurrent enforcing an integer number of flux quanta in the core of the cylinder. Near half-integer flux quanta through the cylinder, the superconductivity is suppressed as it cannot sustain a supercurrent large enough to enforce an integer number of flux quanta through the core. This agrees with recent simulations of the same nanowires [145]. We calculate the effective diameter of the interference loop, assuming half a flux quantum at $B = 0.225$ T, to be $d_{\text{eff}} = \sqrt{2\Phi_0/\pi B} = 76$ nm. As the current density resides inside the nanowire one expects a slightly smaller effective diameter of the electron density than the $\sim 100$ nm diameter of the nanowire. Similar interference effects have recently been observed in full-shell nanowire devices [146, 147].

The qubit behavior can be split in the three lobes separated by the destructive regimes. In the zeroth lobe measured from $B \sim 0$ to $\sim 150$ mT, the device behaves indistinguishably from a standard gate-memon device. Due to the high drive power, mul-
tiphoton transitions are observed exciting higher energy states of the qubit. Around 150 mT the system becomes unmeasurable due to the second qubit on the chip anticrossing with the resonator [Figure B.1]. Figure 7.4 shows Rabi oscillations and lifetime decay at $B = 0$ and $B = 50$ mT. At $B = 0$ we observe lifetimes of $\sim 5.5 \mu s$ similar to previous gate-mon devices verifying that additional plunger gates and dielectrics have not compromised qubit performance. The measurements show almost no difference between $B = 0$ and $B = 50$ mT demonstrating excellent parallel magnetic field resilience consistent with recent studies of gate-mon qubits [113]. Furthermore, as the field is not perfectly aligned it verifies that small out-of-plane magnetic fields do not degrade qubit quality. This might eliminate the need for extensive magnetic shielding required in superconducting qubits.

Moving to higher fields in the first lobe between $\sim 250$ mT and $\sim 650$ mT two main resonances appear. Both states behave as anharmonic oscillator modes with a broad single-photon transition frequency and a sharper two-photon transition separated by $\sim 100$ MHz. While the presence of two anharmonic states is consistent with a large $E_M$ term in the Hamiltonian, it is unlikely that the splitting is due to Majorana physics as one expects higher magnetic fields to enter the topological phase. Rather, the splitting might be connected to low lying energy states coupling to the qubit mode. Indeed several states dispersing strongly with magnetic fields are visible throughout the first lobe. It was not possible to probe the qubit states in time domain due to very low lifetimes.

In the last lobe above $B \sim 650$ mT a single qubit resonance revives with a clear two-photon transition all the way up to $B = 1$ T. Figure 7.5 shows coherent Rabi oscillation of a superconducting transmon qubit at $B = 1$ T with lifetime $T_1 = 0.57 \mu s$ strongly encouraging the feasibility of the experimental setup for hosting Majorana fermions in a coherent transmon. As in the first lobe other resonances strongly dispersing in magnetic field are visible.

### 7.2 Coupled Qubit and Junction states

To further investigate the anomalous qubit-resonance splitting we tune gate electrodes in the first lobe into a regime with sharp transitions exhibiting qubit line splitting shown in Figure 7.6. A clear, uninterrupted qubit transition frequency indicated by a white dashed line is slowly suppressed as the magnetic field is turned up. Additionally, around the qubit transition, several new resonances appear above $B = 350$ mT oscillating as a function of magnetic field. We speculate that these resonances might be explained by an Andreev bound state indicated by the dashed purple line and a second state only weakly dependent on magnetic field around energy $f_0 \sim 6$ GHz as shown by the energy diagram inset [Figure 7.6]. The blue and gray dashed lines are transitions from initially excited states with frequencies $f_0 - f_A$ and $f_0 - f_{\text{qubit}}$ respectively. Purple and blue transitions appear mirrored around $f_0/2$ as their frequencies are given by $f_{\pm} = f_0/2 \pm \delta$ where $\delta = f_A - f_0/2$ (same for white and gray transitions).

At $\sim 360$ mT, an anticrossing appears between the qubit transition and the Andreev transition $f_0 - f_A$ (white and blue lines) indicating a strong coupling. Additionally, the
qubit resonance is also observed uncoupled from the Andreev transition. The coexistence of the coupled and uncoupled spectra might be explained by the odd and even parity of the Andreev state. In the even parity an Andreev state couples to microwave excitation, such as the qubit resonance, while in the odd parity it is uncoupled [148–150]. As the parity of the Andreev state is switching faster than the measurement is performed we observed the average of the two metastable parity states: In the odd parity state, a single qubit resonance (white dashed line) is observed uncoupled from the Andreev state. In the even parity state, two transition frequencies are observed as due to the hybridization of the qubit and Andreev states. As both parity states are measured simultaneously, we observe three resonances in the spectrum. Similarly, when the direct Andreev transition (purple line) is on resonance with the qubit near \( B \sim 390 \) mT three pronounced resonances are observed near 3.3 GHz.

To further probe the spectrum in Figure 7.7 we sweep the gate voltage \( V_{cut} \) at fixed magnetic fields. The strong dispersion of the Andreev state is a signature of a local junction state strongly dependent on the electrostatics around the junction. Guides to the eye indicate the same resonances as in Figure 7.6. As a function of gate voltage clear anticrossings between the qubit resonance and both Andreev transitions are observed. Also in gate voltage, we observe a clear connection between the Andreev transitions given by \( f_\pm = f_0/2 \pm \delta \) further evidence of the simple phenomenological model. At higher fields, more resonances appear complicating a full analysis of the spectrum. Further studies and analysis are necessary to fully describe the multitude of transitions.
CHAPTER 7. HIGH FIELD COMPATIBLE TRANSMON CIRCUIT

Figure 7.6: Two-tone spectroscopy reveals oscillating behavior of junction states at gate voltage $V_{\text{cut}} = -1.805$ V and $V_{\text{pl}} = V_{\text{rl}} = -1.983$ V. Dashed lines are guides to the eye. Inset shows an energy diagram of a phenomenological model consisting of a single strongly dispersing Andreev state $f_A$ and a non-dispersing state $f_0$. An average is subtracted from each column.

7.3 Conclusion

In conclusion, we have presented a simple gate-mon circuit with excellent coherence times of $5 \mu$s at 50 mT. The qubit retains coherence up to magnetic fields of 1 T with lifetime $T_1 \sim 0.6 \mu$s demonstrating the feasibility of the circuit to probe Majorana fermions coherently. At high magnetic fields, we observe several additional resonances in two-tone spectroscopy. We speculate that oscillating transitions present in the spectrum are described by a simple phenomenological model based on Andreev bound states in the nanowire Josephson junction. Further studies with SQUID-type structures, allowing control of the superconducting phase across the Josephson junction, might help to elucidate the origin of these transitions by measuring their energy-phase relations. Alternatively moving to systems with larger $E_C$ would allow distinguishing trivial junction states from anharmonic oscillator modes by measuring charge dispersions, which carries the signature of coupled Majorana fermions as shown in Figure 7.1. Additionally, high-field compatible transmon qubits might open possibilities for hybrid systems such as spin ensemble-based quantum memories in superconducting circuits [151].
Figure 7.7: Two-tone spectroscopy as a function of gate voltage $V_{\text{cut}}$ for different magnetic fields. Dashed lines are guides to the eye. Green dashed line is the resonance of the second qubit coupled to the same readout cavity. An average is subtracted from each column.
Chapter 8

Outlook

It is an exciting time to work in experimental quantum computing research. During the last decade, both existing and emerging technologies for quantum bits have taken tremendous steps towards truly scalable quantum computing. The improvements are powered by new architectures, scalable 2D ion-traps, new ideas, elimination of charge noise in transmon qubits, and new materials, epitaxial semiconductor-superconductor interfaces. The generally agreed upon strategy is to encode qubits in non-local, topological degrees of freedom decoupled from the environment. In this thesis, we have investigated three different approaches towards protected qubits all based on hybrid semiconductor-superconductor nanowires. However, from the studies presented here, or in fact throughout the field of experimental quantum computing, it is impossible to predict which platform will succeed.

In recent years the focus has been on superconducting qubits and quantum error correction. These systems have the advantage of being able to optimize and benchmark single qubits leading to amazing progress. We will likely see the first demonstration of a small scale surface code outperforming its individual parts in the next few years. However, going from small-scale surface code to full-fledged quantum computing is a monumental task. Just the sheer number of qubits required is daunting. On top of that comes a huge amount of classical computing power needed for error correction and signal processing. Fortunately, no single challenge seems unsolvable, and the rest might “just” be engineering. One remaining essential question to be answered in quantum error correction is: Are qubit errors truly local on a large scale?

Topological materials or passive quantum error correction promise a different route to quantum computing based on inherent protection. These platforms will also need a form of active error correction to remove errors, but the expected number of qubits is much lower significantly reducing the challenge of scalability. The trade-off is a complicated material or design problem to create a topological phase. As the topology is created by the system itself it is very hard to prove that the system behaves as it should - any local measurement cannot probe the topological degree of freedom of interest. The main challenge is building the first qubit with control and readout both hard to achieve due to the protected nature. Additionally, the same question needs to be answered as for quantum error correction: Are system errors truly topologically separated from the
protected qubit?

Another tantalizing approach to scalable quantum computing is hybrid systems containing both topological qubits and superconducting qubits. Each type of protection might exhibit different advantages and disadvantages which can be utilized for different parts of a quantum computer. For example, superconducting qubits might be ideal for magic state factories, which needs good initial T-gates, the T-gates performed without protection, as well as fast gate operations for purification. However, as magic state distillation is an inherent random procedure we want to store T-gates in quantum memory until required for computation. Quantum memory requires long-lived qubits ideally with low-overhead of classical computing power - maybe best implemented in low-overhead topological materials. In this thesis, we have demonstrated that the same architecture might host different topological qubits. As topological qubits become a reality, a research direction lies in transferring quantum information from one type of protected qubit to another to take full advantage each platform.

As mentioned it is an incredibly exciting field to be part of, and I look forward to learning about new ideas, inventions, and possibilities along the path to a quantum computer.
Appendices
Appendix A

Second order perturbation theory

In this Section, we explicitly calculate the energy correction to second order in $g/\Delta$ for the coupled circuit of transmon and resonator described in section 3.4. The generalized Jaynes-Cummings Hamiltonian written with product eigenstates $|n, j\rangle$ of $\hat{H}_0$, where $n$ is the resonator excitation and $j$ the atom excitation, is given by:

$$\hat{H} = \hat{H}_0 + \hat{V},$$

$$\hat{H}_0 = \hbar \omega_r \hat{a}^\dagger \hat{a} + \hbar \sum_i \omega_i |i\rangle \langle i|,$$

$$\hat{V} = \hbar \sum_i g_{i, i+1} (\hat{a} |i+1\rangle \langle i| + \hat{a}^\dagger |i\rangle \langle i+1|).$$

The correction to the eigenenergies can be found assuming $g_{ij} = 0$ for $j \neq i \pm 1$. First order correction is

$$E^1_{|n,j\rangle} = \langle n, j | \hat{V} | n, j \rangle = 0.$$
APPENDIX A. SECOND ORDER PERTURBATION THEORY

And second order:

\[ E_{n,j}^2 \equiv \sum_{(m,i) \neq (n,j)} \frac{|\langle m,i|\hat{V}|n,j \rangle|^2}{E_{n,j}^0 - E_{m,i}^0} \]

\[ = \frac{|\langle n+1,j-1|\hat{V}|n,j \rangle|^2}{E_{n,j}^0 - E_{n+1,j-1}^0} + \frac{|\langle n-1,j+1|\hat{V}|n,j \rangle|^2}{E_{n,j}^0 - E_{n-1,j+1}^0} \]

\[ = \frac{g_{j-1,j}^2(n+1)}{\hbar \omega_j - \hbar \omega_r} + \frac{g_{j,j+1}^2 n}{\hbar \omega_r - \hbar \omega_{j+1}} \]

\[ = \hbar \chi_{j-1,j}(n+1) - \hbar \chi_{j,j+1} n \]

\[ E_{n,0}^2 \equiv \sum_{n,i} \frac{|\langle n-1,1|\hat{V}|n,0 \rangle|^2}{E_{n,0}^0 - E_{n-1,1}^0} = -\hbar \chi_{01} n \]

\[ E_{0,0}^2 = 0 \]

where \( \chi_{ij} = g_{ij}^2 / \omega_{ij} - \omega_r \). Collecting the terms the effective Hamiltonian becomes

\[ \hat{H}_{\text{eff}} = \sum_{n,i} E_{n,i} |n,i\rangle \langle n,i| \]

\[ = \hbar \omega_r \hat{a}^\dagger \hat{a} + \hbar \sum_i \omega_i |i\rangle \langle i| + \hbar \sum_i \chi_i |i+1\rangle \langle i+1| \]

\[ - \hbar \chi_{01} \hat{a}^\dagger \hat{a} |0\rangle \langle 0| + \hbar \sum_{i=1} (\chi_{i-1,i} - \chi_{i,i+1}) \hat{a}^\dagger \hat{a} |i\rangle \langle i|, \]

where the resonator states have been written with raising and lowering operators. Lastly, we truncate the transmon to a two-level system:

\[ H_{\text{eff}} = \hbar \omega_r \hat{a}^\dagger \hat{a} + \hbar \omega_1 |1\rangle \langle 1| + \hbar \chi_{01} |1\rangle \langle 1| \]

\[ - \hbar \chi_{01} \hat{a}^\dagger \hat{a} |0\rangle \langle 0| + \hbar (\omega_1 - \chi_{12}) \hat{a}^\dagger \hat{a} |1\rangle \langle 1| \]

\[ = \hbar \left( \omega_r - \chi_{01} |0\rangle \langle 0| + (\chi_{01} - \chi_{12}) |1\rangle \langle 1| \right) \hat{a}^\dagger \hat{a} + \hbar (\omega_1 + \chi_{01}) |1\rangle \langle 1| \]

\[ = \hbar \left( \omega_r - \frac{\chi_{12}}{2} \right) \hat{a}^\dagger \hat{a} + \frac{\hbar}{2} (\omega_1 + \chi_{01}) \sigma_z + \hbar \chi \hat{a}^\dagger \hat{a} \sigma_z, \]

with \( \chi = \chi_{01} - \chi_{12}/2 \) and \( \sigma_z = |1\rangle \langle 1| - |0\rangle \langle 0|. \)
Appendix B

Magnetic field response of NbTiN resonator

Figure B.1 shows resonator response measured interleaved with data in Figure 7.3 for readout frequency adjustments.

Figure B.1: Modulation of resonance frequency of the NbTiN, $\lambda/2$ cavity used for readout in Chapter 7. Due to the large fluctuations, the readout frequency is adjusted each time the magnetic field is move. Large jumps around 0.2 T are due to corrections of the out-of-plane magnetic field on order of $\sim 0.1$ mT (not shown).
Appendix C

Schematics of Experimental Setups

Schematics of each setup used for measurements presented in this thesis. For setups C.2 and C.3 a long cylindrical cryoperm magnetic shield is installed.
Figure C.1: Schematic of setup for single qubit devices in Chapter 5. The data in Figure 5.2 were acquired using a vector network analyzer. For the Sample 1 data in Figures 5.3, 5.4, and 5.5 we mix down to dc and sample the homodyne response, $V_{\text{HH}}$. For the Sample 2 data in Figure 5.5 we mix down to an intermediate frequency before sampling and then perform digital homodyne to extract the cavity phase response.
Figure C.2: Schematic of experimental setup for two qubit device in Chapter 5.
Figure C.3: Schematic of experimental setup device in Chapter 6.
Figure C.4: Schematic of experimental setup for device in Chapter 7. Figure adapted from [144]
Appendix D

Fabrication Recipes

Fabrication recipes for samples presented in this thesis. Metal evaporation and in situ argon milling were done in an AJA International metal evaporation system. E-beam lithography was performed in a 100 kV Elionix electron beam lithography system.

D.1 Single qubit devices presented in Chapter 5

Al film deposition
- Silicon substrate with thermal oxide cleaned in acetone and IPA
- Metal deposition: 1 min Ar mill, 75 nm Al

Gold alignment marks
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Metal deposition: 10 nm Ti, 40 nm Au
- Lift off: Acetone

Al film wet etching
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 μC/cm²
- Etch: 25 s Transene Type D at 54°C, 30 s DI water, 10 s IPA
- Resist strip: Acetone

Nanowire deposition
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Nanowire dry deposition
- Resist strip: Acetone

Gold alignment marks for nanowire
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Metal deposition: 5 nm Ti, 35 nm Au
- Lift off: Acetone
APPENDIX D. FABRICATION RECIPES

Nanowire wet etch

- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 µC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Etch: 12 s Transene Type D at 50°C, 30 s DI water, 10 s IPA
- Resist strip: Acetone

Nanowire contacts and side gate

- Resist spin: EL9, 4000 rpm, 45 s, bake at 185°C for 1 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 µC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Metal deposition: 3 min Ar mill, 1 nm Ti, 150 nm Al
- Resist strip: Acetone

D.2 Two-qubit device presented in Chapter 5

Al film deposition

- Silicon substrate with no thermal oxide cleaned in acetone and IPA
- Metal deposition: 1 min Ar mill, 75 nm Al

Gold alignment marks

- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 3 min
- E-beam exposure: dose 1200 µC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Metal deposition: 5 nm Ti, 45 nm Au
- Lift off: Acetone

Al film wet etching

- Resist spin: EL9, 4000 rpm, 45 s, bake at 185°C for 3 min
- Resist spin: CSAR4, 4000 rpm, 45 s, bake at 185°C for 3 min
- E-beam exposure: dose 450 µC/cm²
- Development: 60 s O-xylene, 120 s MIBK:IPA 1:3, 15 s IPA, O₂ plasma ash
- Etch: 50 s Transene Type D at 53°C, 30 s DI water, 15 s IPA (longer etch time due to contaminated/old etch bottle)
- Resist strip: Acetone

Crossover oxide

- Resist spin: EL13, 4000 rpm, 45 s, bake at 185°C for 3 min
- Resist spin: CSAR4, 4000 rpm, 45 s, bake at 185°C for 3 min
- E-beam exposure: dose 450 µC/cm²
- Development: 60 s O-xylene, 120 s MIBK:IPA 1:3, 15 s IPA, O₂ plasma ash
- Oxide deposition: 250 nm SiO₂
- Lift off: Acetone

Crossover metal

- Resist spin: EL13, 4000 rpm, 45 s, bake at 185°C for 3 min
- Resist spin: CSAR4, 4000 rpm, 45 s, bake at 185°C for 3 min
- E-beam exposure: dose 450 µC/cm²
- Development: 60 s O-xylene, 120 s MIBK:IPA 1:3, 15 s IPA, O₂ plasma ash
- Metal deposition: 3 min Ar mill, 300 nm Al
- Lift off: Acetone
Nanowire deposition
- Resist spin: EL9, 4000 rpm, 45 s, bake at 185°C for 3 min
- E-beam exposure: dose 450 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Nanowire dry deposition
- Resist strip: Acetone

Nanowire wet etch
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Etch: 12 s Transene Type D at 50°C, 30 s DI water, 10 s IPA
- Resist strip: Acetone

Gold alignment marks for nanowire
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 3 min
- E-beam exposure: dose 1200 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Metal deposition: 5 nm Ti, 45 nm Au
- Lift off: Acetone

Nanowire contacts and side gate
- Resist spin: EL9, 4000 rpm, 45 s, bake at 185°C for 1 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 1200 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Metal deposition: 5.5 min Ar mill, 1 nm Ti, 150 nm Al
- Resist strip: Acetone

Al film wet etching
- Resist spin: EL9, 4000 rpm, 45 s, bake at 185°C for 3 min
- Resist spin: CSAR4, 4000 rpm, 45 s, bake at 185°C for 3 min
- E-beam exposure: dose 450 μC/cm²
- Development: 60 s O-xylene, 120 s MIBK:IPA 1:3, 15 s IPA, O₂ plasma ash
- Etch: 20 s Transene Type D at 53°C, 30 s DI water, 15 s IPA
- Resist strip: Acetone

D.3 Device presented in Chapter 6

Deep etch silicon marks
- Resist spin: EL9, 4000 rpm, 45 s, bake at 185°C for 1 min
- Resist spin: CSAR9, 4000 rpm, 45 s, bake at 185°C for 1 min
- E-beam exposure: dose 400 μC/cm²
- Development: 60 s O-xylene, 75 s MIBK:IPA 1:3, 15 s IPA, O₂ plasma ash
- RIE deep etch: Gas cycles C₄F₈:SF₆ 1:1 / C₄F₈
- Resist strip: O₂ plasma

Al film and control etch
- Metal deposition: 1 min Ar mill, 100 nm Al
- Resist spin: AZ1505, 4000 rpm, 45 s, bake at 115°C for 2 min
- UV exposure in a Heidelberg μPG101 LED writer: dose 20 ms
- Development: 60 s AZdev:MQ 1:1, 30 s DI water, 30 s DI water, O₂ plasma ash
- Reactive Ion Etch: ICP 20 s Cl₂, 15 s HBr:Cl₂ 3:5.
- Resist strip: Acetone
APPENDIX D. FABRICATION RECIPES

\[ \text{SiO}_x \text{ crossover insulator} \]
- Resist spin: LOR3B, 4000 rpm, 45 s, bake at 185°C for 5 min
- Resist spin: LOR3B, 4000 rpm, 45 s, bake at 185°C for 5 min
- Resist spin: AZ1505, 4000 rpm, 45 s, bake at 115°C for 2 min
- UV exposure in a Heidelberg µPG101 LED writer: dose 20 ms
- Development: 60 s AZdev:MQ 1:1, 30 s DI water, 30 s DI water, \( \text{O}_2 \) plasma ash
- Oxide deposition: 200 nm \( \text{SiO}_2 \)
- Resist strip: NMP

**Bottom gates,** these got damaged later due to fabrication mistake.
- Resist spin: EL9, 4000 rpm, 45 s, bake at 185°C for 1 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 2 min
- E-beam exposure: dose 1100 \( \mu \text{C/cm}^2 \)
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, \( \text{O}_2 \) plasma ash
- Metal deposition: 30 nm Al
- Resist strip: Acetone

**Nanowire wet etch,** bottom resist layers before nanowire placement meant to protect Al bottomgates from nanowire etch step. This failed leading to an etched bottom gate likely due to a wrong dose in the e-beam exposure. Side gates were added to the design to compensate for the etched bottom gate.
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 2 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 2 min
- Nanowire placement with micromanipulator.
- Resist spin: EL6:A4 2:3, 4000 rpm, 45 s, bake at 185°C for 4 min
- E-beam exposure: dose 500 \( \mu \text{C/cm}^2 \)
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, \( \text{O}_2 \) plasma ash
- Etch: 9 s Transene Type D at 50°C, 30 s DI water, 10 s IPA
- Resist not stripped.

**Nanowire contacts**
- Resist spin: A4, 4000 rpm, 45 s, bake at 115°C for 2 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 115°C for 2 min
- E-beam exposure: dose 1200 \( \mu \text{C/cm}^2 \)
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, \( \text{O}_2 \) plasma ash
- Metal deposition: 5 min Ar mill, 1 nm Ti, 175 nm Al
- Resist strip: Acetone

**Nanowire sidegates**
- Resist spin: A6, 4000 rpm, 45 s, bake at 115°C for 2 min
- Resist spin: A6, 4000 rpm, 45 s, bake at 115°C for 2 min
- E-beam exposure: dose 1200 \( \mu \text{C/cm}^2 \)
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, \( \text{O}_2 \) plasma ash
- Metal deposition: 5 min Ar mill, 1 nm Ti, 175 nm Al
- Resist strip: Acetone

**D.4 Device presented in Chapter 7**

**Tungsten alignment marks**
- Resist spin: LOR3B, 4000 rpm, 45 s, bake at 185°C for 5 min
- Resist spin: CSAR4, 4000 rpm, 45 s, bake at 115°C for 2 min
- E-beam exposure: dose 400 \( \mu \text{C/cm}^2 \)
- Development: 30 s ZED50, 20 s IPA, DI water rinse
- Development: 5 s MF321, DI water rinse, \( \text{O}_2 \) plasma ash
- Metal deposition: 5 nm Ti, sputter \( \sim \)90 nm W
- Lift off: NMP
APPENDIX D. FABRICATION RECIPES

Bottom ALD
- Resist spin: EL13, 4000 rpm, 45 s, bake at 185°C for 1 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 1 min
- E-beam exposure: dose 900 μC/cm²
- Development: 60 s MIBK:IPA 1:3, 15 s IPA, O₂ plasma ash
- ALD deposition: 5 nm HfOₓ at 90°C
- Lift off: NMP

NbTiN deposition and patterning
- Metal deposition: Sputter NbTi in N atmosphere 20 nm
- Resist spin: CSAR9, 4000 rpm, 45 s, bake at 185°C for 2 min
- E-beam exposure: dose 400 μC/cm²
- Development: 60 s O-xylene, 15 s IPA, O₂ plasma ash
- Reactive Ion Etch: PRO ICP etcher with Cl₂ gas
- Resist strip: 1,3-dioxolane

Top ALD
- Resist spin: EL13, 4000 rpm, 45 s, bake at 185°C for 1 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 185°C for 1 min
- E-beam exposure: dose 900 μC/cm²
- Development: 15 nm HfOₓ at 90°C
- Lift off: NMP

Nanowire shell etch and NbTiN crossover insulator
- Resist spin: A4, 4000 rpm with slow acceleration, 45 s, bake at 115°C for 2 min
- E-beam exposure: dose 60 mC/cm² for crosslinked PMMA insulator under NbTiN crossovers
- Development: 60 s MIBK:IPA 1:3, 15 s IPA, O₂ plasma ash
- Etch: 9 s Transene Type D at 50°C, 15 s DI water at 50°C, 60 s DI water
- Resist strip: Acetone

Nanowire contacts and NbTiN crossovers
- Resist spin: A4, 4000 rpm, 45 s, bake at 115°C for 2 min
- Resist spin: A4, 4000 rpm, 45 s, bake at 115°C for 2 min
- E-beam exposure: dose 900 μC/cm² for nanowire shell etch
- E-beam exposure: dose 60 mC/cm² for crosslinked PMMA insulator under NbTiN crossovers
- Development: 60 s MIBK:IPA 1:3, 10 s IPA, O₂ plasma ash
- Metal deposition: 5 min Ar mill, Sputter NbTi in N atmosphere 180 nm
- Resist strip: 1,3-Dioxolane
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